DETERMINAREA DISTRIBUȚIEI FIBRELOR ÎNTR-O SECȚIUNE A UNEI PRISME DIN BETON PRIN ABORDAREA STATISTICĂ ȘI SIMULARE MONTE CARLO. CAZUL SECȚIUNII INFINITE

MONTE CARLO SIMULATION AND STATISTICAL APPROACH OF FIBER DISTRIBUTION IN A SECTION OF A CONCRETE PRISM. INFINITE SECTION CASE

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The aim of this paper is to estimate the number of fibers that cross a section.

Two methods were used, considering a uniform distribution the average number of fibers in any section of a prism with width and thickness considered infinite.

The probabilistic method was used for the central area of the prism, in which the wall effect is not taken into account.

The second method involved a Monte Carlo simulation; the results obtained by the two methods were close.

Experimental validation involved the use of two different dosages, one usual (25kg /m^3) and a much larger one (125kg/m^3), trying to highlight a possible interinfluencing effect of the fibers.

The effect of large granules that could change the distribution of fibers was also studied, by performing concrete samples with only a small 0/4mm aggregate and one with larger 0/16mm aggregates. The experimental results, close to those provided by the two methods, showed a minor influence of the fiber dosage as well as the size of the aggregate granules.

Obiectivul acestei lucrări este o estimare a numărului de fibre care traversează o secțiune.

Au fost utilizate două metode, considerînd o distribuție uniformă numărul mediu de fibre dintr-o secțiune oarecare a unei prisme cu lățime și grosime considerate infinite.

Metoda probabilistică a fost utilizată pentru zona centrală a prismei, în care nu se ține cont de efectul de perete.

A doua metodă a implicat o simulare Monte Carlo; rezultatele obținute prin cele două metode au fost apropiate.

Validarea experimentală a implicat utilizarea a două dozaje diferite, unul uzual (25kg/m^3) și unul mult mai mare (125kg/m^3), încercându-se evidențierea unui posibil efect de interinfluențare a fibrelor.

A fost studiat de asemenea și efectul granulelor mari care ar putea modifica distribuția fibrelor, prin realizarea unor probe de beton doar cu agregat mic 0/4mm și a unuia cu agregate mai mari 0/16mm. Rezultatele experimentale, apropiate de cele prevăzute prin cele două metode, au arătat o influență minoră a dozajului de fibre precum și a dimensiunii granulelor de agregat.

Keywords: *Monte Carlo simulation, fiber, distribution, concrete*

1.Introduction

The number of fibers, their geometrical characteristics, the mechanical ones and the adhesion together with their orientation are the factors that determine the behavior of the composite material to mechanical actions.

This case study aims to estimate the number of fibers in a section of a concrete beam, considered a homogeneous medium, and to compare it with the actual number of fibers obtained in an experiment.

In principle, the dispersion of fibers in concrete should be random. However, this distribution can be influenced by several factors such as: fresh-state properties of the concrete, pouring methods, compaction procedures, wall-effects and formwork geometries. [1].

There are multiple approaches in the literature to the subject of fiber orientation in a matrix of a composite material, both theoretical and experimental [1].

The influence of flowability and wall-effects in the orientation of macro-plastic fibers was studied using the multidirectional double punch test (MDPT) [2]. Cutting of specimen was used to study the influence of compaction procedures [3]. The orientation of fibers in polyolefin fibers reinforced concrete (PFRC) was studied in fractures of samples as a function of technological process of making samples [4]. The orientation of ring type fibers and the number of fibers as a function of fiber geometry, specimen dimensions, and fiber volume fraction in failure plan was studied using fractured surfaces of the samples [5].

Other researchers find that a higher degree of randomness in fibers distribution and orientation is reached when smaller aggregates are used [6].

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Some of them use the finite element method (FEM) which uses the representative volume element (RVE) in order to predict the thermal and mechanical behavior of a fibrous composite subjected to normal and shear stress [7]. Even relatively simple ways to introduce fibers into FEM [8] models have been found.

[9] identifies problems of the two types of existing modelling methods of Steel Fiber Reinforced Concrete (SFRC):

- integral model, which takes into account only simplified fibers contribution into the unified post-peak ductility in concrete element, doesn't reflect the material variation caused by real fibers distribution;

- for the separate model, which consider the concrete matrix and fibers inclusions respectively, the directly randomly fibers distribution still need to be refined. They used X-ray image recognition results from real SFRC material, in order to build a mathematical model of fibers distribution and orientation.

[10] makes a review of existing methods and proposes a model based on the use of coupling finite elements in simulating the behavior of SFRC.

Monte Carlo simulation method was used to describe the tensile behavior of unidirectional fiber reinforced metallic matrix composites [11].

Experimental approaches to fiber distribution use even exotic methods, like X-ray computed tomography which showed a higher degree of randomness in fibers distribution and orientation when smaller aggregates are used [6].

Even special electromagnetic sensor was employed in order to check the number of fibers, their distribution and orientation [12].

Our problem is to find out the number of fibers that cross the middle section of a rectangular parallelepiped concrete beam with dimensions *L* × *l* × *h*, assuming that a number of *N* fibers with length *a* are randomly distributed (see Fig. 1). The orientation of the fibers is random.

Our approach uses a probabilistic estimate of the number of fibers in the section, but also a Monte Carlo simulation as a first step in modeling elastic properties of SFRC.

The number of fibers in a section can be probabilistically estimated using several assumptions:

- the material is homogeneous and does not influence the random distribution of fibers;

- the centers of gravity of the fibers are evenly distributed inside the material;

The Monte Carlo simulation of the experiment considering different equally probable positions of a fiber in relation to the desired section can be carried out with the help of a random or pseudo-random number generator and the Matlab software [13].

Experimentally, the number of fibers crossing a section was determined on 100*100*500mm prisms made with a lower (25kg/m^3) and higher (125kg/m^3) steel fiber dosage and two concrete compositions, one with 0/16mm aggregate and one without large granules, only with 0/4 sand.

These compositions were used to determine if there was a mutual influence of the fibers at high dosages as well as one due to the larger aggregate granules.

The prisms were cut and in the resulting sections the number of fibers was determined, in the central area (70 * 70mm), the side area (70 * 15mm) and the corners (15 * 15mm); the resulting fragments were weighed, then crushed and collected according to EN 14721 + A1: 2008 [14].

From a practical point of view, there is also a problem in the area of the walls, where the fibers cannot take any orientation; the issue will be addressed in a future paper. To avoid this, the fibers will be counted only in the central area of the section.

2. Probabilistic approach

Experimentally, the number of fibers crossing a section was determined on 100*100*500mm prisms made with a lower (25kg/m^3) or higher (125 kg/m^3) fibers dosage.

The problem is practically the three-dimensional version of a classical problem, studied in the 18th century by Georges-Louis Leclerc, Comte de Buffon, considered the first geometric probability problem ever solved [15, 16] :
"Suppose we have a floor made of parallel wooden planks with equal widths L. What is the probability that, by throwing a long needle on the floor, it will intersect a line separating two planks?"
Buffon solved this problem and presented his work in 1777 [17].
In the case of our three-dimensional problem of fibers in concrete beam Lxlxh (see fig.1):
let x be the distance from the center of the fiber to the median plane;
let θ be the angle formed by the fiber with the longitudinal direction of the beam (Figure 2);
the fiber has a chance to intersect the median plane only if $0 \le x \le a/2$;
the probability that the fiber is at a distance less than a/2 from the median plane is $P_1 = a / L$
the probability that the fiber intersects the median plane, as a function of x, when $0 \le x \le a/2$, is the probability that the fiber is inside the spherical sector with angle θ (see Figure 2).
If the radius of the sphere is R = a/2, then the probability is:

$$P(x) = \frac{Vol_{spherical\,sector}}{Vol_{hemisphere}} = \frac{\frac{2\pi R^2}{3}\cdot(R-x)}{\frac{2\pi R^3}{3}} = 1 - \frac{x}{R} = 1 - \frac{2x}{a}$$
(2.1)

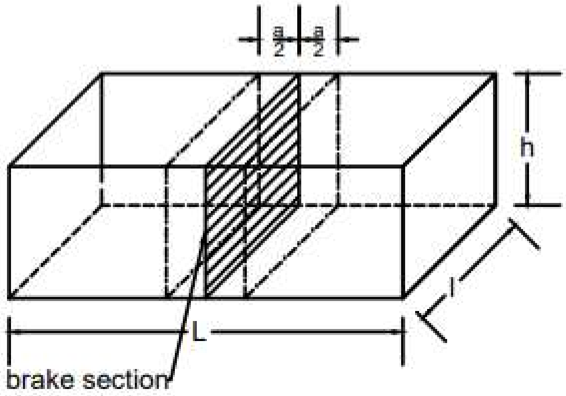

Fig. 1 - Concrete beam with L x l x h dimensions
Grindă de beton cu dimensiunile L x l x h

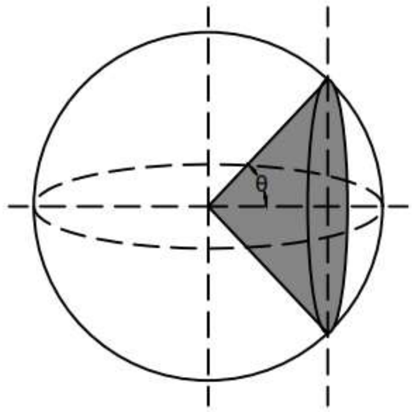

Fig. 2 - Spherical cone of θ angle/*Sector sferic de unghi θ*

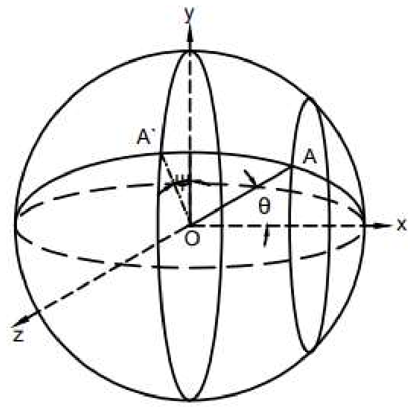

Fig. 3 – The orientation of fiber (continuous line) inside the beam /*Orientarea unei fibre (linia continuă) în interiorul grinzii*

```
function probab = simulareMC(n,L,a)
  H = 0;
  N = 0;
  for i = 1:n
     x = rand*(L/2);
     u = rand*a-a/2;
     v = rand*a-a/2;
     w = rand*a-a/2;
     if u^2+v^2+w^2 <= (a^2)/4
        N = N+1;
        if 4*(u^2+v^2+w^2)*x^2 <= a^2*w*2
           H = H+1;
        end
     end
  end
  probab = H/N
```

Fig.4 – MatLab code for Monte Carlo simulations in concrete beam/*Cod Matlab – simularea Monte Carlo pentru grinda de beton*

Integrating by x and dividing by the length of the integration interval, we obtain the probability that, for 0 ≤ x ≤ a/2, the fiber intersects the median plane:

$$P_2 = \frac{2}{a}\int_0^{a/2} 1 - \frac{2x}{a}\,dx = \frac{1}{2} \tag{2.2}$$

Finally, the required probability is:

$$P = P_1 \cdot P_2 = \frac{a}{2L} \tag{2.3}$$

3. Monte Carlo approach

In the Monte-Carlo method as we intend to use it here, we simulate a large number of random positions of the fiber inside the beam. We then count the number of times the fiber intersects the median plane and we divide it by the total number of random positions. That gives us, as the number of trials increases, an increasingly better approximation of the probability that the fiber intersects the median plane of the beam.

Let the variable *x* be, as before, the distance from the center of the fiber to the median plane. The variable *x* takes random values evenly distributed in the range (0; *L*/2). To describe the orientation of a fiber we will use, similarly to the spherical coordinates, two angles, *θ* and *φ*, as in Figure 3. If we place an *Oxyz* Cartesian coordinate system with origin *O* in the middle of the fiber, with the *Ox* axis along the beam and the plane (*yOz*) parallel to its median plane (*Oy* in the vertical direction and *Oz* in the horizontal direction), then *θ* is the acute angle formed by the direction of the fiber with the *Ox* axis and *φ* is the angle formed by the projection of the fiber on the plane (*yOz*) with the (*Oy* axis (see Figure 3).

In order to be able to correctly apply the Monte Carlo simulation method to this problem, it is critical that the random orientations of a fiber be uniformly distributed in space or, more precisely, on the sphere generated by all possible orientations of the fiber with a certain center. It is worth noting that simply assigning random values to the angle *θ* between 0 and *π*/2 and to the angle *φ* between 0 and *π* does not give us a uniform distribution of fibers orientation, since it gives a much higher concentration of fiber positions close to the north and south poles of the sphere then in the equatorial region.

There are several methods [18] to obtain a uniform distribution of fiber orientations on the sphere, of which the one described below will be used by us to implement Monte Carlo simulations.

To obtain a uniform distribution of the fiber orientations, we first choose a uniform distribution of points in the cube circumscribed to the sphere of radius a/2. This can be done by giving the cartesian x, y, z coordinates of the points (in the Matlab program [13] in Figure 4 denoted by u, v, w random values evenly distributed in the interval [(-a)/2; a/2]. Then we ignore the points that are not inside the sphere, keeping the ones that are. We thus obtain a uniform distribution of points on the inside of the sphere. Each such point corresponds to an orientation of the fiber, and the orientations thus obtained are evenly distributed over the sphere.

In Figure 4 is presented the Matlab [13] code for performing the corresponding Monte-Carlo simulations for this problem. The input parameter n represents the total number of tests, including those corresponding to points outside the sphere. Only the N tests corresponding to points inside the sphere will be considered. The number H of “successful” attempts (the fiber intersects the median plane) is divided by N to obtain the desired probability. We thus have a slight loss of simulation efficiency, because $N \approx 52\% \cdot n$. We must thus use a number *n* practically double to the number *N* of desired Monte Carlo trials . The error of this method is proportional to $1/\sqrt{N}$ [19, 20], which means that in order to increase the accuracy by one decimal place, it is necessary to increase the number of repetitions 100 times.

In this way, for example, a number of $N = 10^6$ needle throws can be simulated in few seconds. In the case of this problem, at this number of simulations, we get an approximation of the result predicted by calculating the probabilities with at least 3 exact decimals.

4. Experimental results and discussions

Hooked steel fibers with a length of 30mm and a diameter of 0.6mm with an average mass of 81,94mg, CEM II B-M 42.5R cement, natural aggregates in the 0/4, 4/8 and 8/16 grades with a density of 2,65Mg/m^3 were used.

Four concrete compositions were made, two with only sand aggregate 0/4 - marked with prefix 4 and two with 0/16 aggregate - marked with prefix 16, in each case the difference between the compositions being the fiber dosage 25 and 125 kg/m^3 respectively - denoted F25 and F125, respectively, according to Table 1.

A laboratory concrete mixer with a capacity of 10L was used in which the aggregates were homogenized with cement, then water and additive and finally the fibers spread by hand over the fresh concrete.

The mixing time was 1 minute for the concrete and then 2 minutes from the addition of the fibers. No fiber agglomerations were observed even for 125kg /m^3 fiber mixtures.

Each mixture was poured into a prismatic mould with a size of 100*100*500mm following the procedure described in EN 14651 [21].

Table 1

The composition of steel fibers reinforced concrete

Compoziția betoanelor armate cu fibre folosite

Dosage/Dozaj (kg/m^3)	**4F25**	**4F125**	**16F25**	**16F125**
Cement/ Ciment	350	350	350	350
Water/ Apa	166	166	155	155
Admixture/, Aditiv	4.2	4.3	2.8	3.0
Sand/nisip 0/4	1818	1784	838	823
Gravel/sort 4/8	0	0	372	366
Gravel/sort 8/16	0	0	652	640
Fibers/ Fibre	25	125	25	125

Table 2

Fiber mass distribution in concrete slices 100*100mm/

*Distribuția masei fibrelor pe feliile 100*100mm din prisma din beton*

Slice number/ numarul sectiunii	**Fiber mass/Masa fibrelor (g)**			
	4F25	**4F125**	**16F25**	**16F125**
1	6.93	32.58	7.57	37.40
2	6.48	34.74	7.14	47.55
3	7.00	34.97	7.11	43.93
4	7.83	36.24	8.98	41.53
5	7.76	33.35	8.26	42.48
6	7.08	36.88	6.13	42.35
7	6.64	41.37	7.57	45.39
8	7.29	43.06	8.01	39.35
9	7.60	39.76	8.64	42.23
10	8.91	36.53	8.48	40.84
11	9.05	26.06	7.27	34.49
12	8.91	37.12	7.31	33.95
13	7.96	35.44	7.29	28.90
14	6.47	35.88	7.01	36.97
m_{med} (g)	7.57	36.00	7.63	39.81
St dev	0.89	4.07	0.77	5.02
Cv (%)	11.80	11.31	10.04	12.60

The consistency of the mixtures was determined according to EN 12350-2 [22] and was corresponding to class S2 in all cases, in order not to favor fiber segregation.

The 100*100*500mm prisms were kept in moulds for 3 days after which they were cut into 30mm transverse strips (fiber length) starting from 20mm at one end. For each strip, the fibers in the 70 * 70mm central area of the section were counted in order to avoid the wall effect.

The mass of the fibers in each slice of prism having the same thickness can be used to evaluate the homogeneity of their distribution; the results are presented in Table 2.

Kolmogorov-Smirnov test [23] for detecting a normal distribution of results was carried out for every fibers mass of each slide and the result showed that conditions were met in every case.

Minor variations of the coefficient of variation Cv for fibers masses (as seen in Table 2) are observed both in the case of the variation of the maximum size of the aggregate and of the fiber dosage. It seems that mixes have the same degree of homogeneity.

The results of the fiber counting in the 70*70mm concrete sections are presented in Table 3.

Kolmogorov-Smirnov test [23] applied to 14 values series showed a normal distribution of results.

The coefficient of variation Cv (as seen in Table 3) show smaller and quite closed values for high dosages of fibers and somewhat higher and sensible different for different aggregates sizes.

Table 3

Number of fibers counted on 70*70mm concrete slices surfaces
*Numărul de fibre numerate în secțiuni de 70*70mm*

Slice number/ *Numarul sectiunii*	**N**			
	4F25	**4F125**	**16F25**	**16F125**
1	17	112	30	122
2	21	156	23	155
3	26	148	31	127
4	17	135	18	187
5	23	105	13	183
6	15	136	16	192
7	22	133	22	138
8	18	137	31	143
9	21	122	9	159
10	33	84	23	140
11	22	127	29	129
12	16	119	23	134
13	16	113	22	83
14	18	91	12	133
Nmed	20.4	122.7	21.6	144.6
St dev	4.85	20.47	7.22	29.10
Cv (%)	23.8	16.7	33.4	20.1

For the statistical approach in the case of a fiber of length *a*=30mm with an average mass of 81,94 mg, of a prism length of L = 0.5m, with the dosage D of fibers, in its central area (70 * 70mm) are found the number of fibers (N) given by the following relation:

$$N = D * L * 0.07 * 0.07 / 0{,}081944 \qquad (4.1)$$

Using equation 2.3 it can be found the number estimated of fibers in the section, presented in Table 4. It can be seen a good estimation of number of fibers in the section for small dosages of fibers (25kg/m^3) and a under evaluation of the number for high dosages (125kg/m^3). The explanation for this result must lie in the fact that our test section is not infinite and represents in fact 49% of the prism section. Indeed, it is possible that with the higher dosage of fibers, the wall effect leads to pushing of more fibers towards the center of the prism section, resulting in a higher count of fibers in the test region than probabilistically expected.

Table 4

Number of fibers in a central 70*70mm column 500mm long of the beam, evaluated number of fibers in a slice and average number found

*Numărul de fibre dintr-o coloană cu secțiunea de 70*70mm și lungimea de 500mm alături de numărul evaluat de fibre precum și numărul mediu efectiv găsit într-o felie de 30mm*

Fiber dosage/ Dozaj fibre (kg/m^3)	N	N_{stat}	N_{MC}	4F	16F
25	747	22,4	22,3	20,4	21,6
125	3737	112,1	111,7	122.7	144.6

Monte Carlo approach show no significant variations with the number of simulations: 500.000, 1.000.000 and 2.000.000 and even in the standard deviations for the same numbers when repeated 10 times, as it can be seen in Table 5. Meantime, the values are very close to those obtained in probabilistic approach. The same under evaluation of fiber number in concrete section can be seen for high dosages.

For both approaches the evaluated values are close to the average value determined experimental in an interval equal to $X_m \pm \sigma$.

Table 5

Arithmetic mean and standard deviation for 10 simulations and different numbers of trials/ *Valoarea medie și abaterea standard pentru câte 10 simulări ale unor numere diferite de încercări*

Simulation run no.	N=500000	N=1000000	N=2000000
Mean	0.03006	0.02996	0.02991
Standard deviation	0.00032	0.00021	0.00013

5. Conclusions

It was estimated by two methods the average number of fibers in any section of a prism with width and thickness considered infinite, considering a uniform distribution of fibers.

The probabilistic method is a 3D version of Buffon's problem and was relatively easy to solve in this case; for the other cases, of the areas close to the walls and corners, the mathematical treatment is significantly more difficult.

The Monte Carlo simulation provides a valuable investigative tool and can be applied in this case as well, the results by the 2 methods being very close, as expected.

The experimental verification took into account two possible influencing factors, namely the fiber dosage and the size of the aggregate granules.

A good match of the experimental results was found with those predicted theoretically, especially in the case of lower fiber dosages (25kg/m^3). In the case of higher dosages (125kg/m^3), a slight theoretical underestimation was found compared to the experimental results, which can be attributed to the wall effect that appears in the real experiment, but it's not accounted for in the theoretical approach.

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