STRENGTH ASSESSMENT OF LIGHTWEIGHT CONCRETE CONSIDERING METRIC VARIANCE OF THE STRUCTURAL ELEMENTS

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Macrostructure of the modified expanded clay concrete in the matrix-filler form system was considered. Statistical criteria of the macrostructure expanded clay concrete elements using a topological composition and fractal invariants were determined. The criteria describe metric discrepancy of length, diameter and area of macrostructure elements with their topological equivalents. It is shown that taking into account the indicators of metric discrepancy between length of the interphase and intraphase diameter boundaries, pore and coarse aggregate area leads to accuracy increase and is consistent with the theoretical prerequisites for influence of the investigated structural elements on strength. It has been established that the empirical model, taking into account the metric area deviation of expanded clay gravel fractions, has the highest accuracy in concrete strength predicting (R^2=0,92), and the model for flexural strength assessing taking into account the diameter of its fractions has the lowest accuracy (R^2=0,44). As the study results have shown, the statistical criteria for metric discrepancy of structural elements can be used as correction coefficients or constants in existing empirical models for strength assessing of lightweight concrete, taking into account the macrostructure effect.

Keywords: *concrete, structure, strength, fractal dimension, metric discrepancy, topological equivalent, model.*

1. Introduction

Concrete compressive strength is a key characteristic reflecting its performance [1, 2]. In most cases, the prediction of the concrete strength properties, in particular lightweight concrete, based on the study of their morphology and on the statistical results of the structural components assessment. However, the results of concrete quality criteria forecasting are influenced by many parameters of their mixing technology. Even a slight change in the technological process can lead to a significant change in the concrete quality [3, 4]. Therefore, modeling the structure and properties of many materials, according to the theory of K. Gödel [5], may be incomplete formal axiomatics. According to S. Beer [6], the arising incompleteness of the formal axiomatics can be partially compensated by the search and application of more adequate approaches to identify the object under study.

To partially eliminate the incompleteness of the formal axiomatics that arises when assessing the structure effect on the lightweight concrete strength properties, it is proposed to use the fractal geometry of B. Mandelbrot [7] with topological approach. Prospects for the application of fractal formalism in concrete science are considered, for example, in [8], where fractal analysis of the binder particles characteristics, surface texture of aggregates, pore structure in concrete, viscosity and energy of destruction was carried out; coarse cement pastes with fly ash [9]; the thermal conductivity of building materials was determined [10], etc. The fractal approach is used to rank the physical and mechanical properties of expanded clay concrete based on the analysis of the areas of their self-similarity [11], the assessment of quality criteria for multi-parameter technologies [12, 13], and the solution of incorrect problems in materials science [14]. A distinctive feature of the fractal geometry application in the analysis of materials structural elements with various geometric complexity is an arbitrary method of specifying the metric to select their adequate approximation.

The properties of expanded clay concrete, for thin-walled floating reinforced concrete structures, have been studied. [15]. To study the effect of the expanded clay concrete structure on its strength, the authors propose to take into account the metric variance between geometric characteristics of the structure elements (length, diameter, area) and their topological equivalents, taking into account the fractal dimension of the elements. Approximation by topological equivalents was first applied by K. Smith describing the processes of steel microstructure formation [16]. For example, the symbiosis of topological and fractal invariants for assessing the metals structure made it possible to reduce the forecast error of strength and plasticity indicators by 1,24 ... 2.16 times, depending on the grade of metal.

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Reference metal structures were used in [17] as topological equivalents.

In this work, due to the lack of a unified standard scale of diameters and areas for describing various structural elements of lightweight concrete, as well as length of interphase and intraphase boundaries, their average values were taken as the topological equivalent. The dimension of the topological equivalent E on the concrete studied plane sample was taken equal to its Euclidean dimension.

The metric deviation calculation of the expanded clay concrete structural elements with a topological equivalent was carried out using the following statistical criteria:

- The metric difference in the *element sizes* of the phase structure was estimated by the value of the variance δ_d between the diameter of the topological equivalent of the phase d and the diameter of the phase studied elements d_g, taking into account their fractal dimension D_d (1):

$$\delta_d = \sqrt{(d^E - d_g^{D_d})^2} / d^E \tag{1}$$

- metric discrepancy in the length of interphase, intraphase boundaries and voids was estimated by the variance value δ_l between the boundaries length of the topological equivalent l and the boundaries length of the studied elements l_g taking into account their fractal dimension D_l (2):

$$\delta_l = \sqrt{(l^E - l_g^{D_l})^2} / l^E \tag{2}$$

The "tortuosity" of interphase and intraphase boundaries in (2) was taken into account by the value of their fractal dimension.

- metric discrepancy δ_S between *cell area* of the topological equivalent S and elements area of the real structure S_g, taking into account their fractal dimension D_S, was determined by the formula (3):

$$\delta_S = \sqrt{(S^E - S_g^{D_S})^2} / S^E \tag{3}$$

The rationale for choice of these criteria can be the fact that the real concrete structure elements have a complex geometric configuration. It is assumed that the more "deformed" boundaries of the concrete structure elements, the greater the difference in their geometric characteristics with topological equivalent. Thus, the more shape of the structure element differs from the shape of the topological equivalent, the greater difference between topological and fractal dimensions [17]. Probably, the failure to take into account this fact partially explains existing discrepancy in the results of predicting physical and mechanical properties of lightweight concrete using only the Euclidean structure characteristics.

2.Materials and methodology

In order to study the influence of the elements macrostructure metric divergence on the strength properties of expanded clay concrete, 15 concrete mixtures were studied (Table 1). The concrete was prepared on Sulphate resistance Portland Cement 400 CEM I 32.5 R/SR. The cement content within the limits of experiment varied from 500 to 600 kg/m^3; sand – from 595 to 665 kg/m^3; gravel – from 645 to 665 l/m^3; silica fume – from 0 to 50 kg/m^3. The amount of S-3 plasticizer was 0,5...1% of the cement mass (density 2,5...6 kg/m^3). As a coarse porous aggregate, we used claydite gravel grade 150 with fractions diameter d=5...10 mm and bulk density 600 kg/m^3. The fine aggregate was washed quartz sand with a fineness modulus 2,7. The concrete mixing technology was selected taking into account the results obtained in the study of the porous aggregate effect on the lightweight concrete properties [15]. The concrete mixture was prepared in a forced mixer while loading components in the following order: water with admixture, cement, expanded clay gravel, sand. After loading expanded clay gravel and before loading sand, the mixture was mixed for one minute. The technological process provided pre-treatment of porous gravel with a cement suspension at the initial stage of mixing. The total mixing time was ~ 5 minutes.

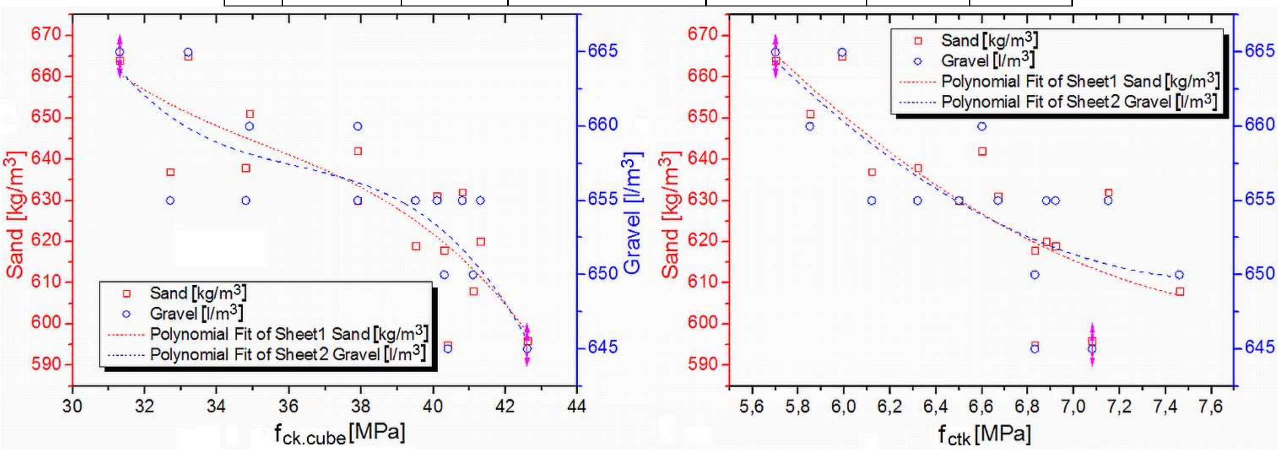
Despite the significant influence of the concrete compositions on the quality criteria, it is not always possible to establish a correspondence between them, since the concrete manufacture technology is multi-parameter [18, 19]. A number of other factors affect concrete quality indicators (mineralogical composition of cement, dispersion of cement particles, preparation method, etc.) [3]. It is rather difficult to take into account the complex influence of these factors within one model framework. Therefore, the working area of technology parameters usually corresponds to a property field, which is not always represented by linear models with high correlation coefficients. In this case, this is confirmed by the dependence of the concrete compressive strength $f_{ck.cube}$ on the sand and gravel content, which is described by third degree polynomials with pair correlation coefficients for sand $R^2=0,65$ and for gravel $R^2=0,50$ (Fig. 1 *a*). The dependence of the concrete flexural strength on the composition is described by polynomials of the second degree with indicators $R^2=0,64$ (for sand) and $R^2=0,49$ (for gravel) (Fig. 1 *b*).

Samples for testing all 15 expanded clay concrete compositions had a typical structure shown in Fig. 2. Among the main elements of the macrostructure are following: 1 – claydite gravel; 2 – cement-sand matrix; 3 – grains of feldspar in

Table 1

The compositions of the investigated expanded clay concretes

No	Cement (kg/m^3)	Sand (kg/m^3)	Claydite gravel (l/m^3)	Silica fume (kg/m^3)	S-3 (kg/m^3)	Water (l/m^3)
1	500	664	665	0	2.5	214
2	500	665	665	0	5.0	169
3	500	651	660	25	3.75	183
4	500	637	655	50	2.5	219
5	500	638	655	50	5.0	188
6	550	642	660	0	4.13	175
7	550	630	655	25	2.75	197
8	550	631	655	25	4.13	178
9	550	632	655	25	5.5	161
10	550	618	650	50	4.13	196
11	600	619	655	0	3.0	209
12	600	620	655	0	6.0	194
13	600	608	650	25	4.5	188
14	600	595	645	50	3.0	235
15	600	596	645	50	6.0	191



a b

Fig. 1 - Influence of sand and gravel content on expanded clay concrete strength $f_{ck.cube}$ (*a*) and f_{ctk} (*b*)

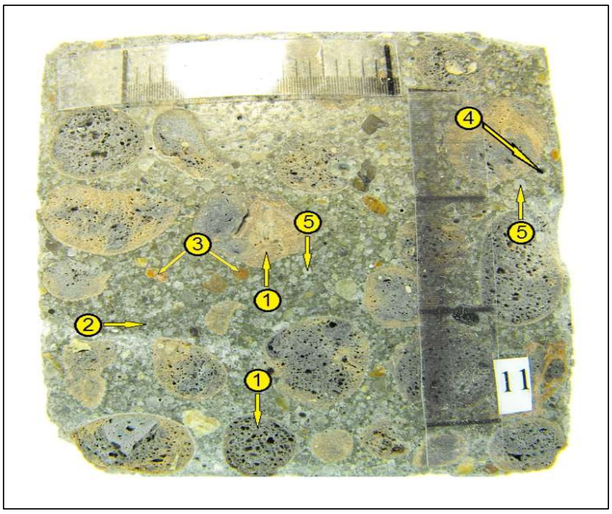


Fig. 2 - Concrete macrostructure (experimental point № 11)

the sand; 4 – large and small pores; 5 – coarse quartz fractions.

To calculate the fractal dimensions *D* of the macrostructure elements, a patented technique was used [20]. The technique is based on searching for the convergence of the fractal dimension numerical values, determined using the cellular and point methods. An example of calculating the cell and point pores dimensions (Fig. 2) is shown in Fig. 3.

The bilogarithmic dependence of the cells number *N(l)*, covering the object of study, depending on the cell size *l*, is shown in Fig. 3 *a*. Fractal dimension D_{bdm} corresponds to the pore boundaries dimension; $D_{tone.k}$ and $D_{tone.t}$ – cell and point dimensions of pores, respectively; $D_{background.k}$ and $D_{background.t}$ – cell and point dimensions for other objects, which are automatically taken as background objects. Fig. 3 *b* it follows that the best convergence for the cellular and point fractal pores dimensions was fixed at the ninth step of the iterations *Dt=1,844 (l=9)*. The best convergence of the background objects dimensions was determined at the fifth step of the calculations. *Df =1,908 (l=5)*.

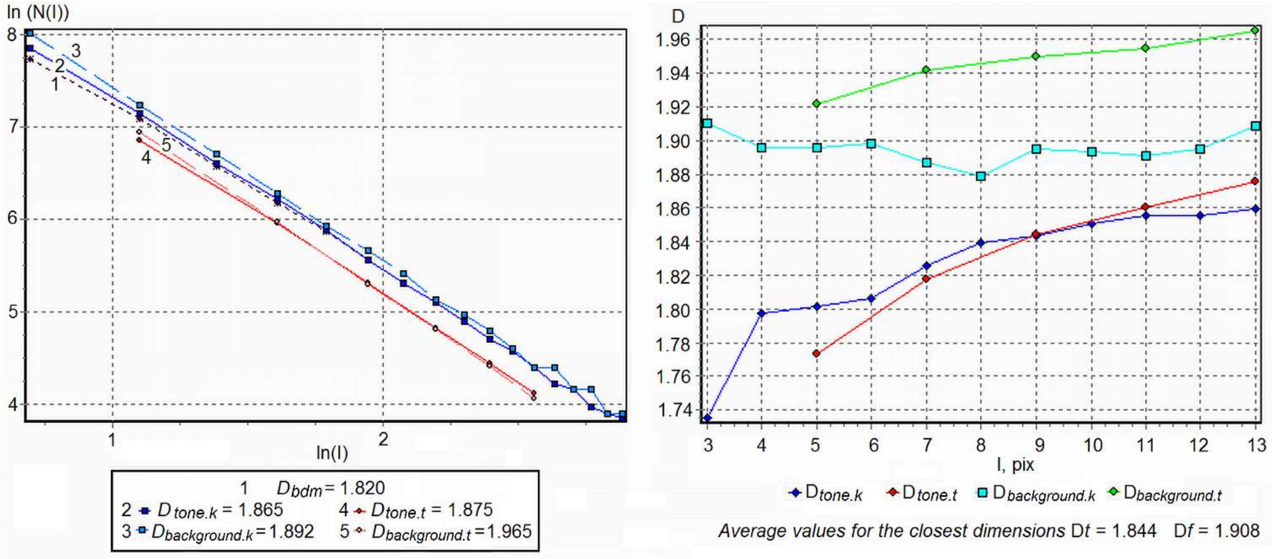


Fig. 3- Calculating stages of the fractal pores dimension.

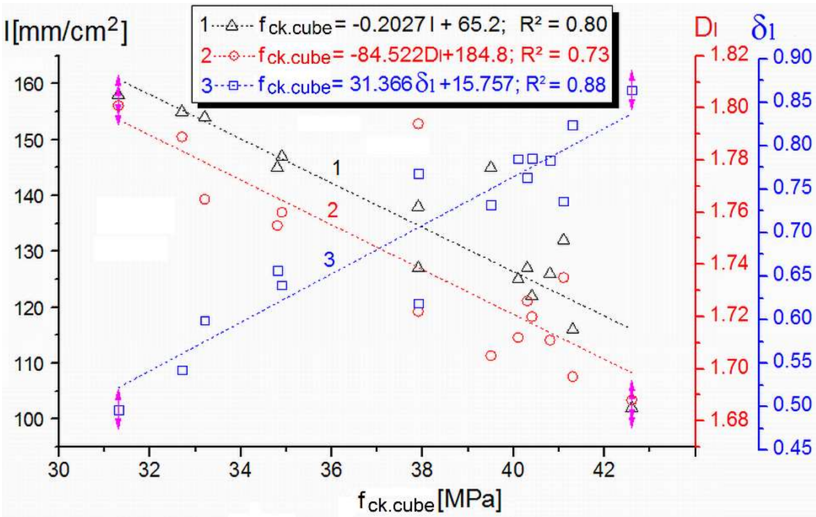


Fig. 4 - Relationship between strength $f_{ck.cube}$ and length of interphase and intraphase boundaries l (1), their fractal dimension D_l (2), metric length discrepancy δ_l (3)

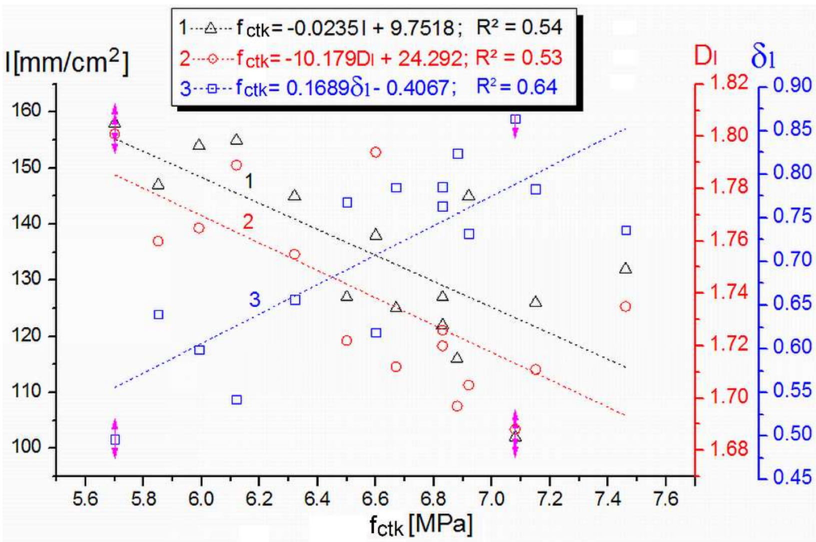


Fig. 5 - Relationship between strength f_{ctk} and length of interphase and intraphase boundaries l (1), their fractal dimension D_l (2), metric length discrepancy δ_l (3)

3. Discussion of experimental results

The expanded clay concrete macrostructure was considered, which is a matrix-filler system with numerous micro- and macro-voids located in a cement-sand matrix, aggregates and contacts between them (interphase and intraphase boundaries). Taking into account the metric discrepancy between the interphase length and intraphase boundaries of macrostructure elements and the pore boundaries length δ_l made it possible to reduce the relative error of the strength prediction models from 3,26% (dependence 1) to 2,40% (dependence 3), shown in Fig. 4. At the same time, a decrease in the fractal dimension of the boundaries structural elements length and pores D_l from 1,801 to 1,688 was recorded with an increase in the concrete compressive strength from 31,3 to 42,6 MPa (dependence (2) in Fig. 4). The error in predicting the concrete flexural strength according to model (1) (Fig. 5) was 3,75%, and using the model for predicting strength by metric length discrepancy (3) (Fig. 5), the error decreased to 3,50%. This is consistent with the fact that the boundaries of contacts between the concrete macrostructure elements are usually defects in the form of discontinuities and microvoids, which reduce the strength indicators. For example, in [3] it is indicated that with a volume of intergranular voids of 5%, the lightweight concrete strength decreases by 20% more compared to concrete of a dense structure, which indicates the importance of taking them into account when predicting strength.

An increase in the length of the structural elements boundaries l leads to an increase in their fractal dimension D according to the Mandelbrot relation [7]:

$$l \sim \delta^{D}, \qquad (4)$$

where: δ – the link boundary size of the element under study selected during iteration.

Separately, the issue of the voids boundaries length contribution to the change in the strength indicators of expanded clay concrete was considered. Calculations have shown that the dependence of strength $f_{ck.cube}$ on the interphase and intraphase boundaries length without taking into account the length of voids in the form of pores is described by a polynomial of the second degree with a low correlation coefficient R^2=0,29. Taking into account the pore length into the total length of interphase and intraphase boundaries leads to an increase in the prediction of strength indicators (Fig. 6). At the same time, the average error ε in predicting the expanded clay concrete compression strength without taking into account the boundaries length of the voids l^* was 7,45%, and taking into account the boundaries length of the voids l decreased to 3,26%. The average error ε in predicting the concrete flexural strength without taking into account the boundaries length of the voids also exceeded the error taking into account the boundaries length of the voids: 5,73 % > 3,73 %.

In addition, when studying the effect of the grain boundaries length and voids boundaries on the strength properties of expanded clay concrete, their "tortuosity" was taken into account using fractal dimension D_l. This ultimately made it possible to determine the influence of the metric discrepancy in the length of the boundaries δ_l macrostructure elements of the expanded clay concrete for strength properties $f_{ck.cube}$ and f_{ctk}.

The results of calculating the metric length discrepancy δ_l, diameter δ_d and area δ_S of the investigated macrostructure elements of the investigated 15 expanded clay concrete compositions with their topological equivalents are shown in Table 2.

When studying the influence of diameter d, area S and fractal dimension D of large and small pores on the strength index $f_{ck.cube}$ of expanded clay concrete, the correlation coefficients of the obtained linear models varied in the range 0,62…0,69 (Fig. 7 *a*). Large pores had a diameter of up to 1 mm, and their content was in the range of 2...7%. The diameter of small pores varied from 0,3 to 0,5 mm with a content of 3...4%. The total content of large and small pores in concrete did not exceed 11 %. During the experiment, it was found that the fractal dimension of the pores decreased linearly with their diameter and increased with increasing pore area. The simultaneous increase in the dimension and pore area is consistent with the results [21], where *S(D)* dependence is also described by a first degree polynomial with $R^2=0,72$. Models describing the relationship between the concrete flexural strength and metric discrepancies in the diameter δ_d and area δ_S the pores have close correlation coefficients R^2=0,74 and R^2=0,71 (Fig. 7 *b*).

The models for estimating flexural strength using the values of the pore area and their metric discrepancy have a higher prediction accuracy compared to the other models in Fig. 8. The dependences $f_{ctk}(S)$ and $f_{ctk}(\delta_S)$ also have close values of the correlation coefficients 0,72 and 0,74, which may indicate the possibility of their equivalent use in determining strength indicators. The values of the models coefficients $f_{ctk}(d)$ and $f_{ctk}(\delta_d)$ are also close – 0,61 and 0,62 respectively.

The results of modeling the strength properties of expanded clay concretes indicate their sensitivity to changes in the characteristics of metric discrepancy in the length of interphase and intraphase boundaries, metric discrepancy in the diameter and pores area.

The paper investigated the effect of expanded clay gravel grains on the strength properties (Fig. 9 and Fig. 10). Gravel grain diameters varied from 5 to 10 mm, mostly had a round slightly elongated shape. The gravel content varied from 38 to 42%. Models for assessing strength indicators

Table 2

Characteristics of structural elements and properties of expanded clay concrete

№	Properties		Claydite gravel, $5 \le d \le 10$ mm					Large and small pores, $0,3 \le d \le 1$ mm					interphase and intraphase boundaries taking into account the pore length		
	$f_{ck.cube}$, MPa	f_{ctk}, MPa	*d*, mm	*S*, %	*D*	δ_d	δ_S	*d*, mm	*S*, %	*D*	δ_d	δ_S	Σl, mm/cm^2	D_l	δ_l
1	31.3	5.7	8.50	42.0	1.96	0.03	0.06	0.90	6.8	1.95	0.51	3.32	158	1.80	0.50
2	33.2	6.0	8.10	42.0	1.96	0.05	0.05	0.85	7.0	1.91	0.36	3.31	154	1.77	0.60
3	34.9	5.9	8.30	42.0	1.95	0.02	0.08	0.80	6.5	1.88	0.22	2.89	147	1.76	0.64
4	32.7	6.1	8.30	41.5	1.97	0.01	0.05	0.85	6.2	1.90	0.36	2.25	155	1.79	0.54
5	34.8	6.3	8.00	41.0	1.95	0.10	0.14	0.80	5.0	1.85	0.23	1.25	145	1.76	0.66
6	37.9	6.6	8.40	41.0	1.94	0.04	0.17	0.80	5.6	1.87	0.22	1.45	138	1.79	0.62
7	37.9	6.5	8.00	41.0	1.95	0.10	0.13	0.60	6.0	1.80	0.26	0.47	127	1.72	0.77
8	40.1	6.7	7.90	39.5	1.93	0.15	0.24	0.70	4.9	1.68	0.02	0.65	125	1.71	0.79
9	40.8	7.2	7.80	39.0	1.94	0.16	0.24	0.65	4.5	1.82	0.16	0.72	126	1.71	0.78
10	40.3	6.8	7.75	39.0	1.94	0.18	0.25	0.70	5.0	1.82	0.03	1.15	127	1.73	0.76
11	39.5	6.9	8.00	40.0	1.94	0.12	0.21	0.70	3.5	1.84	0.04	0.04	145	1.71	0.73
12	41.3	6.9	7.60	38.0	1.94	0.20	0.27	0.72	5.0	1.72	0.05	0.61	116	1.70	0.82
13	41.1	7.5	8.10	38.0	1.95	0.08	0.26	0.65	2.7	1.66	0.09	0.43	132	1.74	0.74
14	40.4	6.8	7.50	39.0	1.94	0.22	0.24	0.60	2.4	1.64	0.20	0.54	122	1.72	0.79
15	42.6	7.1	7.70	38.0	1.95	0.17	0.26	0.70	3.0	1.62	0.04	0.96	102	1.69	0.86
topological equivalents			8.00	40.1	-	-	-	0.73	4.9	-	-	-	135	-	-

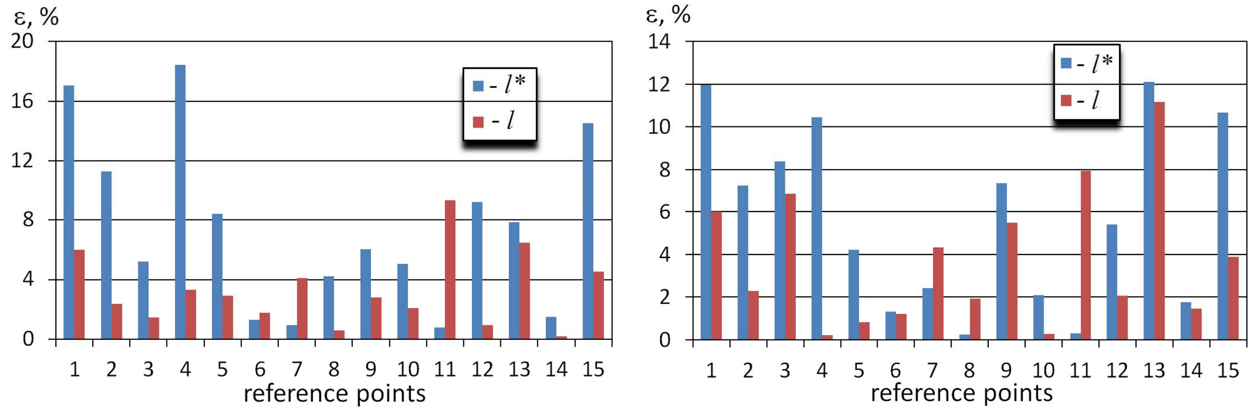


Fig. 6 - Distribution histogram of relative errors ε strength prediction $f_{ck.cube}$ (*a*) and f_{ctk} (*b*) without taking into account the boundaries length of voids *l** and taking into account their length *l*

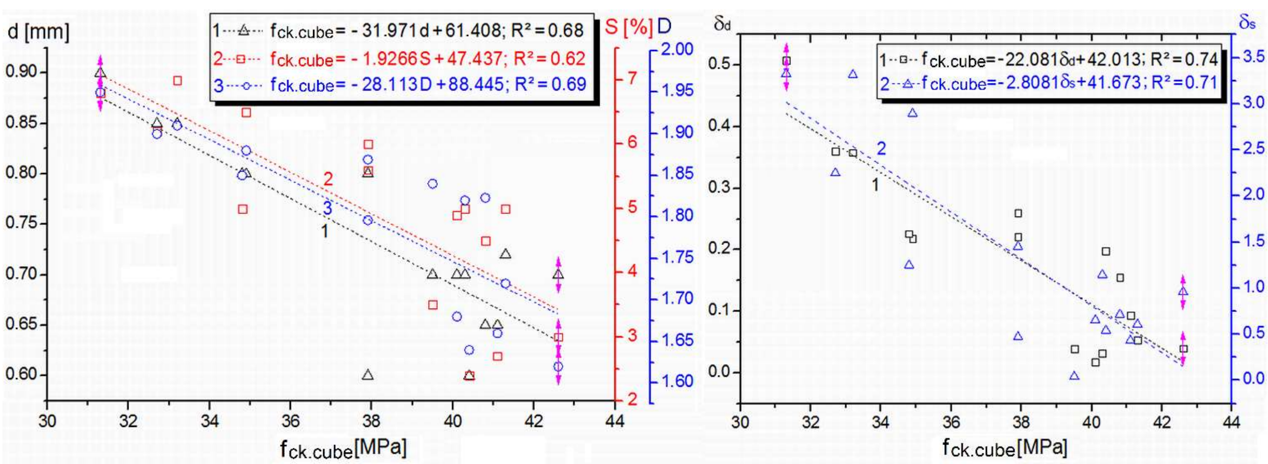
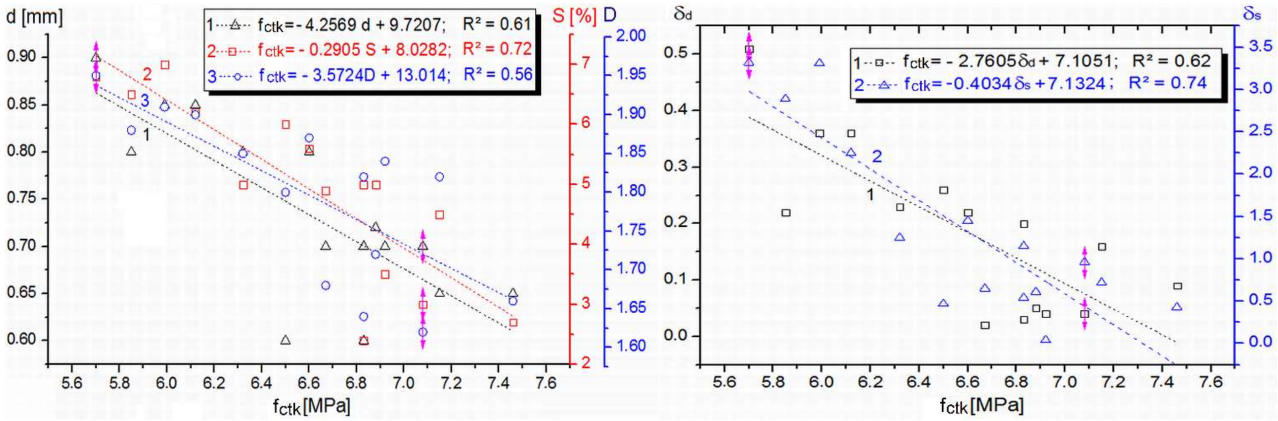


Fig. 7 - Strength relations $f_{ck.cube}$ with Euclidean and fractal characteristics of pores (*a*); metric discrepancies in pores diameter and area (*b*)



a b

Fig. 8. - Strength relations f_{ctk} with Euclidean and fractal characteristics of pores (*a*); metric discrepancies in pores diameter and area (*b*)

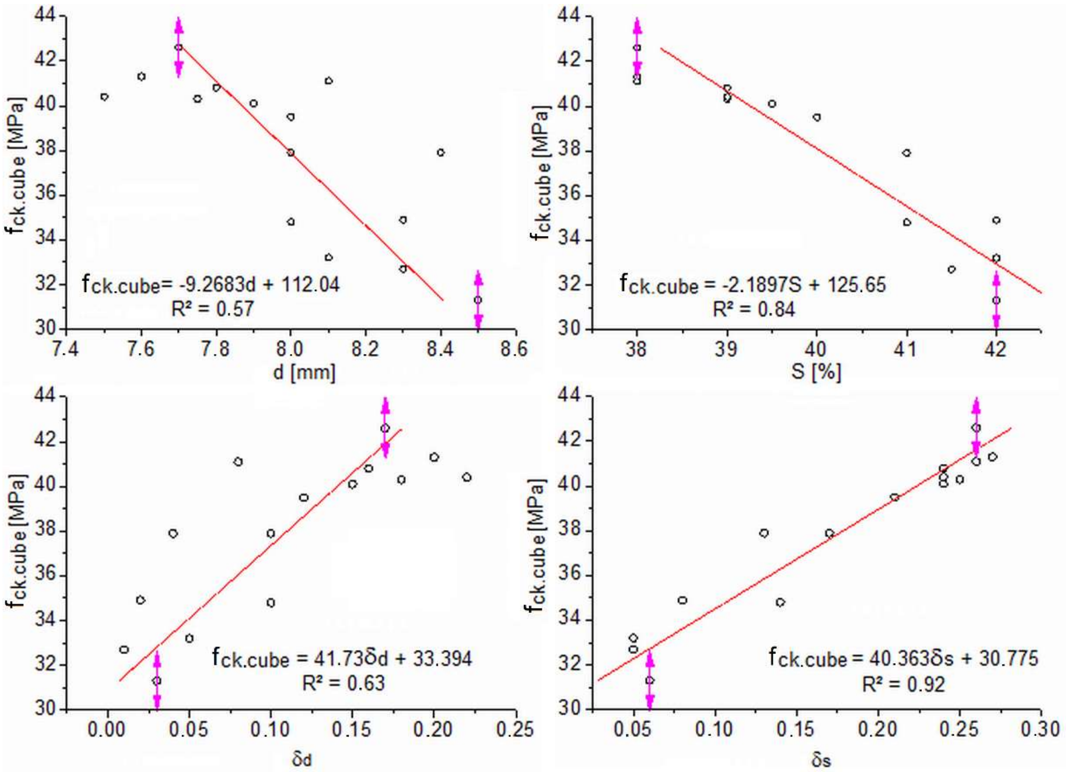


Fig. 9 - Strength ratio $f_{ck.cube}$ with Euclidean characteristics of expanded clay gravel and their metric discrepancies

based on the characteristics of metric discrepancy in the area and sizes of expanded clay gravel grains are superior in accuracy to models for assessing strength based on Euclidean characteristics (Fig. 9). With a decrease in the aggregate grains size, the strength properties increase simultaneously with an increase in their metric discrepancy. Difference in the metric divergence of the macrostructure elements with the topological equivalent characterizes the degree of structure heterogeneity, which has a significant effect on a number of physical and mechanical concrete properties [15].

The difference in strength assessment models f_{ctk}, based on the Euclidean characteristics of the diameter and area of expanded clay gravel grains with their metric discrepancies is insignificant (Fig. 10). For example, R^2 for the first degree polynomial $f_{ctk}(S)$ was 0,83, and for $f_{ctk}(\delta_S)$ – 0,82. These results are explained by the obtained close to Euclidean values of expanded clay gravel fractal dimensions, which varied in the range 1,93...1,97 (Table 2) due to their round elongated shape. The shape grains configuration influenced the insignificantly changing metric discrepancy of their diameter with the topological equivalent; the difference between the values was 4%. Therefore,

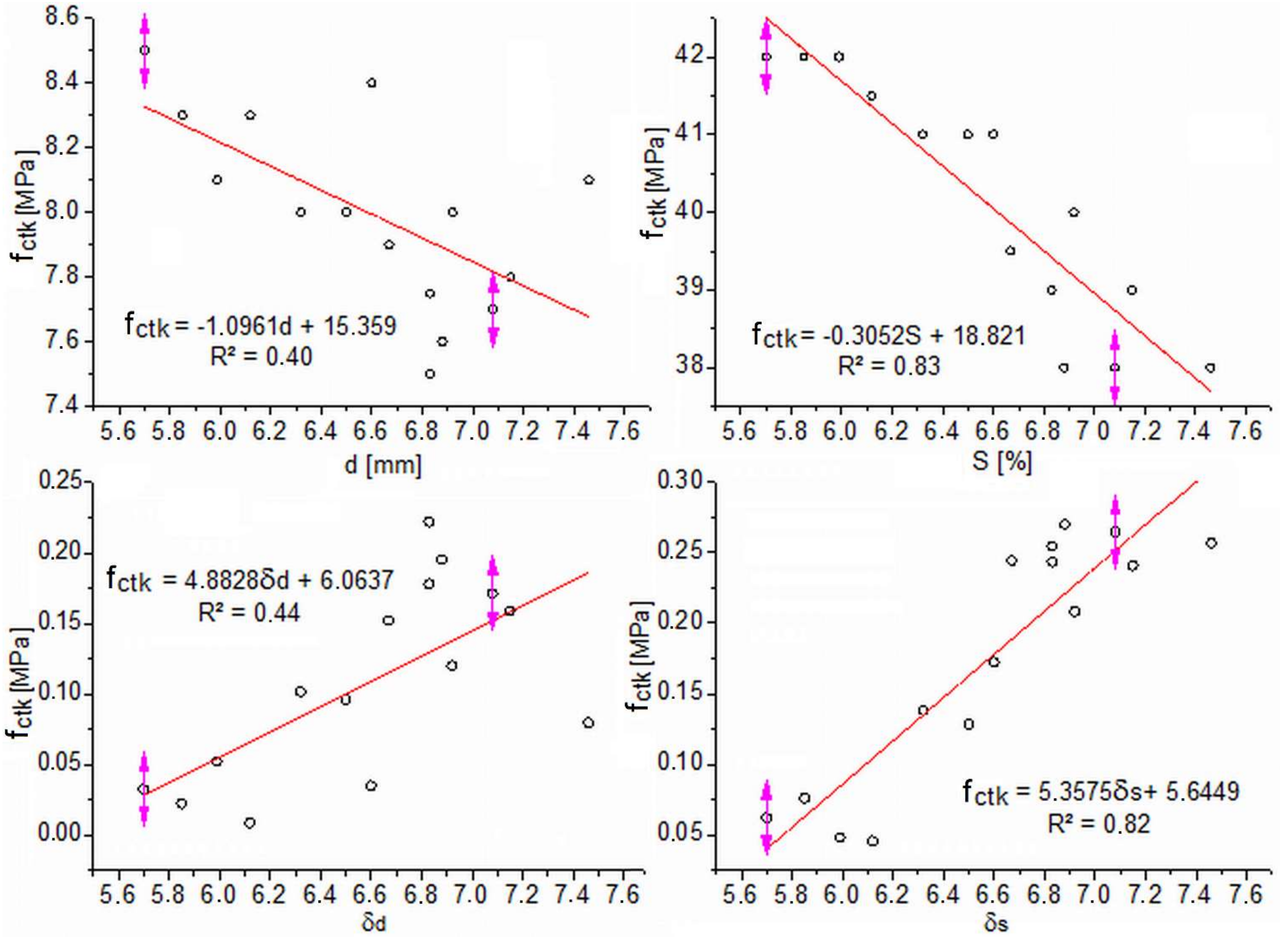


Fig. 10 - Strength relations f_{ctk} with Euclidean characteristics of expanded clay gravel and their metric discrepancies

in this case, it is preferable to assess the strength f_{ctk} in terms of the expanded clay gravel area S, a decrease in which has a positive effect on strength.

A comparative results analysis of the assessing strength indicators $f_{ck.cube}$ and f_{ctk} using the Euclidean macrostructure characteristics of the expanded clay concrete and their metric discrepancy with topological equivalents is carried out. In most of the considered cases, taking into account the metric discrepancy between the length of interphase and intraphase boundaries, diameter, pore area and expanded clay gravel leads to an increase in the forecast of strength properties. The metric divergence of structural elements is partly explained by the complex structure of the real concrete structure, which in most cases is formed in open systems under no equilibrium thermodynamic conditions, and therefore it can be described using the fractals theory [22].

4.Conclusion

The paper investigates the influence of the metric discrepancy between the length of the interphase and intraphase boundaries, size and area of the large aggregate pores (expanded clay gravel) on the expanded clay concrete strength. As a result of the data analysis, the following conclusions can be drawn:

(1) It was found that the metric discrepancy between the length of the interphase and intraphase boundaries δ_l more effectively characterizes concrete compressive strength $f_{ck.cube}$ (R^2=0,88), than flexural concrete strength f_{ctk} (R^2=0,64).

(2) Taking into account the contribution of the voids boundaries length to the total length of interphase and intraphase boundaries makes it possible to reduce the average forecast error $f_{ck.cube}$ from 7,45 % to 3,26 %, and the forecast error f_{ctk} from 5,73 % to 3,73 %.

(3) The influence of air pores on the concrete flexural strength is less than on the compressive strength, which is confirmed by a comparative models analysis of their metric diameter divergence δ_d, and is consistent with the results of the study [23].

(4) The significant effect of expanded clay gravel on the concrete compressive strength was confirmed by taking into account the metric discrepancy in the area of its fractions (R^2=0,92). Concrete flexural strength is well described by the model of the metric divergence values of the fractions area of expanded clay gravel (R^2=0,82) and by the model describing the effect of its area (R^2=0,83).

(5) Within the experiment framework, fractal dimension of the interphase and intraphase

boundaries length changed by 6,11% (1,69...1,80), the dimension of the pores changed by 16,9% (1,62...1,95), the dimension of the expanded clay gravel fractions changed by 2,03% (1,93...1,97) due to their regular rounded shape. Obtained results indicate that the concrete macrostructure elements have fractal properties.

The obtained results indicate the prospects for using statistical criteria in the framework of models for predicting strength properties of lightweight concrete, which are based on the metric discrepancy of the length, diameter and area of real structural elements with their topological equivalents. Also, taking into account the statistical criteria of macrostructure elements in the design of expanded clay concrete compositions opens up an additional possibility of optimizing mixtures, which will make it possible to obtain concretes with predictable properties at a lower cost for the experiment.

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