

# COMPORTAREA GRINZILOR DIN BETON ARMAT SOLICITATE ÎN REGIM DINAMIC ÎN DIVERSE STADII DE DEGRADARE

## BEHAVIOUR OF REINFORCED CONCRETE BEAMS SUBJECTED TO DYNAMIC LOADING IN VARIOUS STAGES OF DEGRADATION

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*The dynamic characteristics of a building element are represented by vibration frequency, damping coefficient and stiffness. The vibration frequency of a reinforced concrete element modified with respect to the degradation state caused by the cracking of concrete and the yielding of reinforcement is analyzed in this paper.*

*An experimental program, carried out at the Faculty of Civil Engineering and Building Services in Iasi, revealed a specific relation between the vibration frequency and the damage level of some reinforced concrete elements. In this experimental program four reinforced concrete beams with the same concrete class but different reinforcing ratios have been tested in bending up to various levels of degradation. The experimental results have been validated through numerical modeling using the software package LUSAS based on finite element analysis.*

*Caracteristicile dinamice ale unui element de construcții sunt caracterizate prin frecvența proprie de vibrație, coeficientul de amortizare și rigiditatea acestuia. În lucrare se studiază variația frecvenței proprii de vibrație a unor elemente din beton armat în raport cu starea de degradare cauzată de fisurarea betonului și curgerea armăturii.*

*Un program experimental, efectuat la Facultatea de Construcții și Instalații din Iași, a evidențiat relația dintre frecvența proprie de vibrație și stadiul degradării unor console din beton armat solicitate la încovoiere. În cadrul programului au fost testate, până la diverse stadii de degradare, patru grinzi din beton armat cu aceeași clasă de beton dar cu armări diferite. Rezultatele experimentale au fost validate prin modelare numerică utilizând programul pe bază de element finit LUSAS.*

**Keywords:** reinforced concrete beams, vibration frequency, damage level, numerical modeling

### 1. Introduction

In any structural system the presence of degradation leads to a stiffness variation and subsequently to the change of its vibration frequency. These arguments and the possibility to measure vibrations conducted to the development of several methods for identification of structural degradation. Many case studies relating to this topic have been presented and analyzed in the specialized literature [1-6].

The problem of identifying the degradations, through the measurement of the dynamic characteristics, is difficult because it depends on the quantity and quality of the experimental data.

If only the basic aspects are followed in the case of a concentrated degradation located in a construction element with a linear behavior before and after the appearance of the degradation is considered, it can be modeled as an opened crack

or by modifying the stiffness mechanical properties of the element.

Consistent with the usual techniques of monitoring, a structural degradation can be quantified through two characteristics: its location and the stiffness reduction of the affected cross-sections. The most effective form is the one in which the stiffness of the entire undegraded structure is known, since it can be analyzed with respect to the stiffness of the structure determined at a given moment.

The vibration frequency of a reinforced concrete element is modified directly proportional with the degradation state. The degradation state is defined by the cracking of concrete and the yielding of reinforcement.

An extensive experimental program was performed at the Faculty of Civil Engineering and Building Services in Iasi to determine a relation between the vibration frequency and the damage level of several reinforced concrete beams [7].

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## 2. Experimental program

### 2.1. Identification of the degradations

The identification of a degradation in an element is simple and well represented when the degradation position is known. The opposite situation, which consists in determining the degradation parameters when the modal parameters (vectors, structure frequency) are known, is much more complex.

To identify a number  $r$  of cracks it is compulsory to know  $2r$  self-frequencies. This aspect is very useful in the process of choosing the model to identify the degradations.

In general, the identification procedure of the degradation parameters, based on the dynamic measurements, consists of:

- determination of the structural model in undamaged conditions;
- determination of the structural model with degradations.

Through the comparison of the two models the degradation parameters can be assessed.

In the analysis the structural model with degradations is considered, because the undegraded model is adopted as being known [8]. The problem is severely restricted by the accuracy of the experimental data and of the utilized degradation model.

A beam with a length equal to  $L$  and a rectangular cross-section  $w \times h$ , having one or more cracks, Figure 1, is selected to illustrate the working procedure, [9].

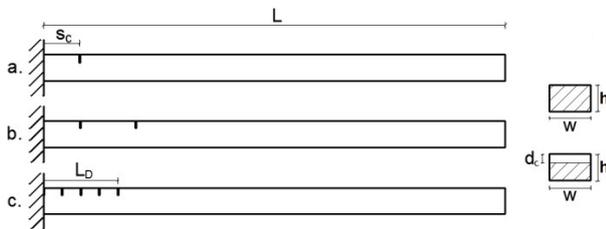


Fig. 1 - Damaged beam model / Model grindă degradată:

- a. with a single crack / cu o singură fisură; b. with multiple independent cracks / cu fisuri multiple independente; c. with multiple cracks which interact / cu fisuri multiple ce interacționează

It is assumed that the cracks have a small opening, the same height and a constant depth, denoted  $d_c$ , along the width of the beam. The cracks do not influence the mass of the element.

When there is a small crack, Figure 1a, the degraded model is represented by two parameters: position of crack and damage level.

The degradations corresponding to the multiple cracks can be analyzed using the model of those with only one crack but considering a hinge with rotation stiffness for each crack. If the multiple cracks are independent, Figure 1b, they will be localized at a bigger distance than the conventional length of the affected area. When the cracks are

multiple and interact between them, Figure 1c, the region in which they appear will have a lower stiffness. The last case is typical for the reinforced concrete elements.

The degradation model is characterized by three parameters: location,  $s$ , extension,  $b$ , and damage level,  $d$ , determined as follows:

$$s = \frac{S_c}{L} \quad (1a)$$

$$b = \frac{L_D}{L} \quad (1b)$$

$$d = 1 - \frac{EI^D}{EI} \quad (1c)$$

where:

- $s$  – location;
- $b$  – extension of the damaged area;
- $d$  – damage level;
- $S_c$  – length, according to Figure 1;
- $L$  – length of the element;
- $L_D$  – length of the damaged area, according to Figure 1;
- $EI$  – the flexural stiffness modulus in undamaged state;
- $EI^D$  – the flexural stiffness modulus in damaged state.

### 2.2. The outline of the experimental testing

Four reinforced concrete beams have been tested in the experimental program. The beams presented the following characteristics:

- length  $L = 2.60$  m;
- transverse cross-section  $0.15 \times 0.30$  m;
- concrete cover  $a = 0.025$  m;
- concrete class C12/15;
- reinforcement at the inferior part: two longitudinal plain bars made of steel S235 [10] with the nominal diameter of 6 mm;
- reinforcement at the superior part:
  - beams G1 and G2: two longitudinal ribbed bars made of steel S355 [10] with the nominal diameter of 12mm, Fig. 2;
  - beams G3 and G4: three longitudinal ribbed bars made of steel S355 with the nominal diameter of 12mm;
- transverse reinforcement: stirrups made of steel S235 with the nominal diameter of 6mm, positioned at the distance of 150mm.

Due to the significant influence on the final results of the curing conditions of the elements, the beams have been prepared in accordance to the standard recommendations [11].

The testing stand was built so that the structural model of the reinforced concrete beams was a cantilever. A force transducer with a capacity of 50 kN and a displacement transducer with a wire and an accelerometer, has been utilized for the data acquisition system,

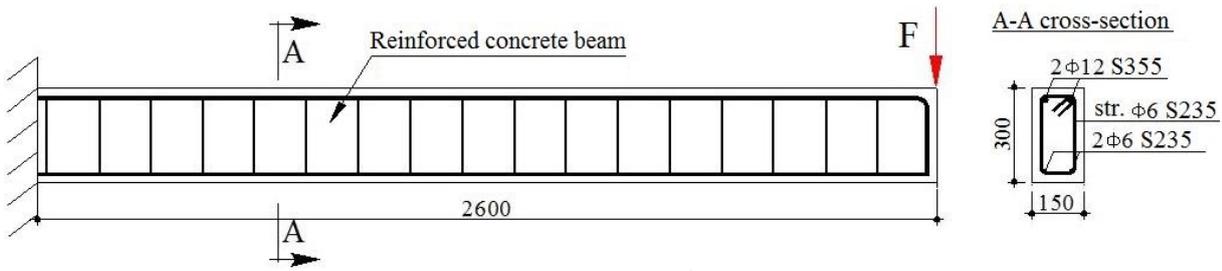


Fig. 2 - The reinforced concrete element composition, beam G1/G2 / Alcătuirea elementului din beton armat, grinda G1/G2.

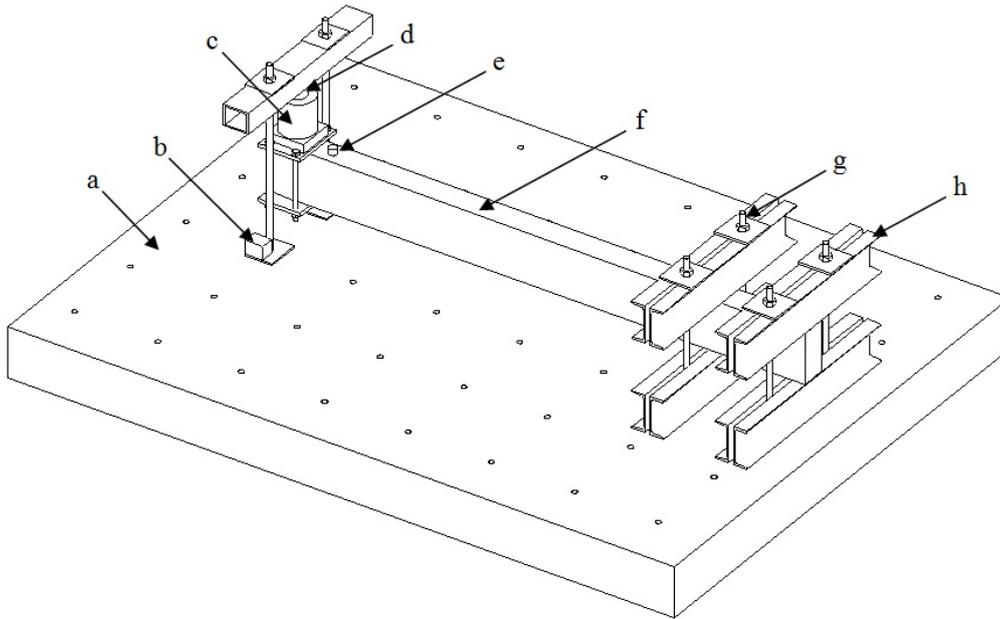


Fig. 3 - Outline of the experimental testing / Schema de ansamblu a încercării experimentale: a. concrete stand / platformă din beton; b. displacement transducer / traductor de deplasare; c. hydraulic jack / cric hidraulic; d. force cell / traductor de forță; e. accelerometer / accelerometru; f. reinforced concrete beam / grindă din beton armat; g. bolts / buloane; h. steel profiles / profile metalice.



Fig. 4 - a. Beam G1 loaded until failure / Grinda G1 solicitată până la cedare; b. Crack pattern of G1 beam / Ansamblu de fisuri în grinda G1.

positioned as shown in Figure 3. During the test, between the beam and the loading transducer metallic plates have been positioned so the loading on the free end of the element was uniformly applied.

### 2.3 Testing method

The beams have been subjected to bending by applying a static concentrated increasing load in the free end until failure, Figure 4.

The testing was performed in two stages:

- degradation of the element through the application of a load from top to bottom using a hydraulic jack that can measure the

force and the displacement in the direction of the applied load;

- excitation of the free end of the cantilever beam using a rubber hammer; the response was recorded with an accelerometer and a data acquisition system.

### 2.4. Experimental results

The dynamic characteristics of the reinforced concrete cantilevers have been determined on several loading steps corresponding to the different damage level of the elements.

The first loading step is the one in which the element is loaded only from its self-weight. Due to the reduced tensile strength, the concrete cracked under the own weight of the cantilever, and the element had, from the beginning, a damage level equal to 0.469.

The damage level is evaluated according to the flexural stiffness modulus in the undamaged state (determination based on ultrasound measurements) and the flexural stiffness modulus in the degraded state. The response of the system collected by the accelerometer is taken over by the acquisition system, converted to a digital signal and recorded on the computer, as illustrated Figures 5 and 6. The sampling interval is equal to 0.005 seconds with a maximum time period of 20.48 seconds corresponding to 4096 values. The progression of the dynamic characteristics for different stages of degradation is analyzed in detail for the G1 beam.

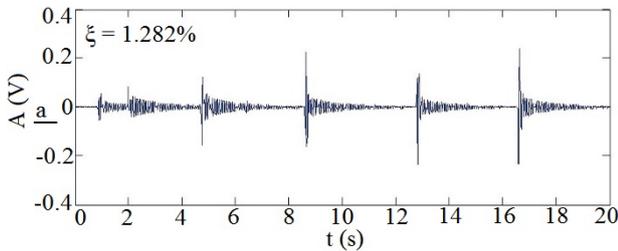


Fig. 5 - Time-domain response for the G1 beam. Loading step: 0 kN / Răspuns în domeniul de timp pentru grinda G1. Treapta de încărcare: 0 kN

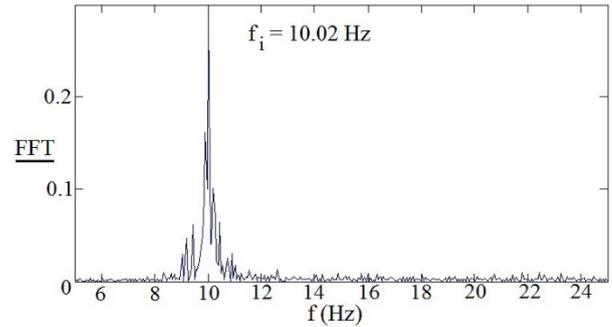


Fig. 6 - FFT analysis for the G1 beam. Loading step: 0 kN / Analiza FFT pentru grinda G1. Treapta de încărcare: 0 kN.

The action of the impulse given by the rubber hammer can be observed in Figure 5. On the graphic display this is represented by a maximum of the acceleration, which has a damped decrease in approximately 15 seconds.

A frequency of 12.35 Hz was obtained for the first loading step, using the Fast Fourier Transform (FFT) analysis, Figure 6. The frequency determined analytically for an undegraded state was 19.98 Hz. It can be observed that the first cracks have a very high influence on the element stiffness and on the vibration frequency.

In the loading step equal to 2 kN, the concrete in the tensioned area continues to crack

Table 1

Summarization of the experimental results / Centralizarea rezultatelor experimentale

	Load [kN]	Displacement [mm]	Frequency [Hz]	Damping [%]	Damage level
Beam G1	0	0	12.35	1.282	0.469
	2	4	11.57	1.702	0.547
	3	6	11.13	1.966	0.556
	4	8	10.79	1.859	0.561
	6	16	10.7	1.883	0.664
	8	23	10.5	1.912	0.692
	10	30	10.02	2.054	0.707
	11.8	92	8.59	1.566	0.885
Beam G2	0	0	12.21	1.464	0.403
	2	4	11.67	1.587	0.531
	4	8	10.89	1.925	0.552
	6	15	10.79	1.877	0.64
	8	22	10.55	1.588	0.676
	10	35	10.45	1.687	0.745
	12	110	9.28	1.953	0.907
Beam G3	0	0	12.35	1.393	0.438
	2	4	12.26	1.434	0.539
	4	8	11.18	1.746	0.556
	6	15	11.13	1.835	0.642
	8	22	11.13	1.612	0.677
	10	31	10.9	1.528	0.715
	12	39	10.49	1.824	0.73
Beam G4	14	115	9.18	2.053	0.891
	0	0	12.4	2.227	0.424
	2	4	11.62	2.905	0.536
	4	8	11.13	3.28	0.555
	6	15	11.08	2.385	0.641
	8	22	10.89	2.325	0.677
	10	30	10.45	1.923	0.706
	12	40	10.25	2.307	0.736
14.5	111	8.59	2.148	0.884	

and the frequency of the element decreases from 12.35 Hz to 11.75 Hz. The increase in the damage level, from 0.469 up to 0.547, is directly proportional with the decrease of the element frequency.

In the third loading step, the concrete in the tensioned area cracks even more, the damage level increases up to 0.556, the critical damping fraction reaches 1.966 and the characteristic frequency drops to 11.13 Hz.

In the loading step of 4 kN, the stresses in the reinforcement increase, but do not reach yielding yet. New cracks appear in the tensioned area of the concrete. The critical damping fraction has a small decrease compared to the previous step. The characteristic frequency descends to 10.79 Hz and the damage level increases to 0.561.

In the loading steps from 6 kN to 8 kN the element behavior is similar to the previous step. The critical damping fraction slightly increases up to the value of 1.912%, the characteristic frequency decreases up to 10.5 Hz and the damage level reaches 0.692.

At values of the load between 10 kN and 11.8 kN the reinforcement reaches yielding. The flexural stiffness of the element has a significant decrease and the element is deformed. The critical damping fraction records a small increase for the 10 kN step and then decreases. The frequency of the element descends to 8.59 Hz, and the damage level reaches 0.885. For an easier way to track the results obtained, the values from the experiments have been summarized in Table 1.

Failures of the beams G1 and G2 occurred at approximately 12 kN, while the beams G3 and G4 failed at a higher load, of approximately 14 kN.

The characteristic frequencies have values between 8.59 Hz and 12.4 Hz and decrease with respect to an increase in the damage level of the element. The critical damping fraction has values between 1.282% and 3.28%, with a minimum value corresponding to the first loading step, then it increases until the reinforcement reaches yielding, moment when it decreases again.

By analyzing the load-displacement curves shown in Figure 7, in the first loading steps it can be noticed a higher flexural stiffness that decreases gradually, until the reinforcement reaches yielding, and at the end has a sudden drop until failure. The beginning of the reinforcement yielding can be easily distinguished on the graphs.

In Figure 8 the variation of the element frequency with respect to the loading step, which causes degradations in the element, can be observed. In the first loading steps, up to 6 kN, the frequency has an almost linear variation. In this stage, the concrete in the tensed area cracks but it still has a contribution on the flexural stiffness of the element. Between the loading step of 6 kN and up to the yielding of the reinforcement, the concrete in the tensed area has no more influence,

the load being taken only by the reinforcement. In this region, the frequency has a nonlinear decrease with respect to the loading. After the yielding of the reinforcement, the relation between frequency and loading step is still linear but it has a diminished slope.

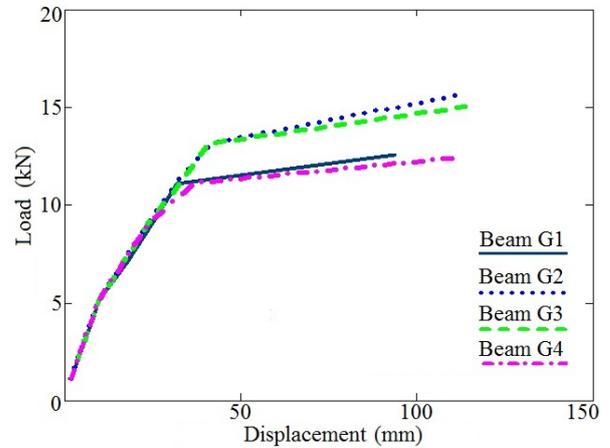


Fig. 7 - Load-displacement graph for the beams G1 - G4 / *Grafic încărcare-deplasare pentru grinzile G1 - G4.*

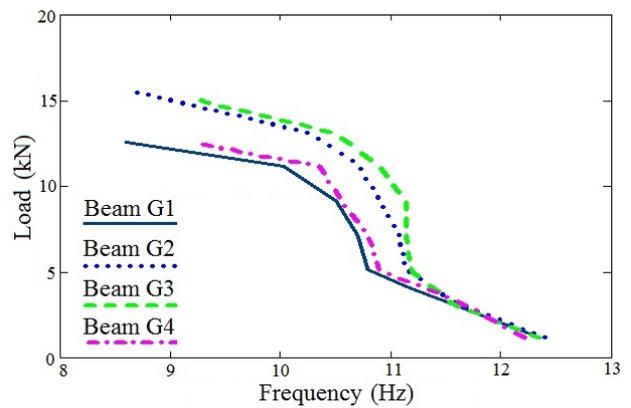


Fig. 8 - Load-frequency graph for the beams G1 - G4 / *Grafic încărcare-frecvență pentru grinzile G1 - G4.*

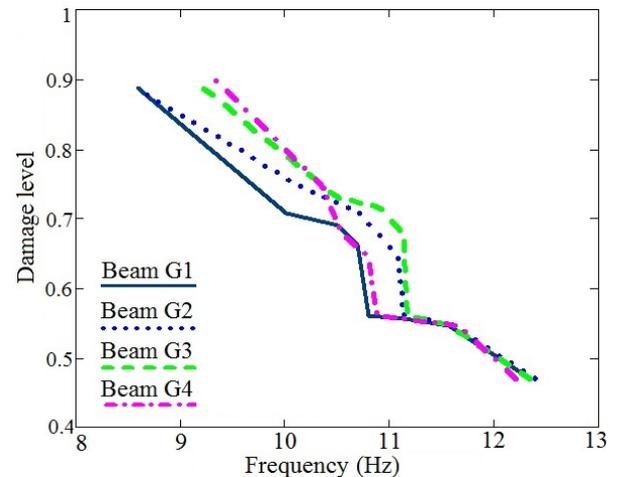


Fig. 9 - Damage level-frequency graph for the beams G1 - G4 / *Grafic grad de degradare-frecvență pentru grinzile G1 - G4.*

From the analysis of the experimentally obtained damage level - frequency graph of the reinforced concrete elements, presented in Figure 9, it can be noticed that at a more reduced damage level, up to approximately 0.55, the frequency has a linear variation with the damage level with a diminished slope. This aspect underlines the fact that for a reduced damage level the change in frequency is significant. At a damage level between 0.55 and 0.7 the frequency is maintained almost constant. A damage level of approximately 0.7 corresponds to the beginning of the reinforcement yielding. At a damage level of over 0.7 the frequency has a linear behavior with respect to the damage level but with a lower slope (frequency is slightly modified with respect to the damage level).

### 3. Numerical modeling

#### 3.1. The numerical analysis of the reinforced concrete cantilever

The study presents the elasto-plastic numerical analysis of a reinforced concrete element, fixed at one end (cantilever), for different loading steps corresponding to various damage levels up to failure. The numerical analysis was performed with the program LUSAS based on finite element analysis. The program has the option to consider the reinforcement nonlinearity and the generation of cracks in the concrete.

#### 3.2. Element composition

The model built for the finite element analysis maintains the same geometrical shape, sizes, properties of the materials, reinforcement and structural model with the experimentally tested beams. Considering the similar behavior of the beams G1 - G4 in the numerical analysis only the case of the beams G1 - G2, reinforced with 2Ø12 S355 was performed.

#### 3.3. The materials characteristics

The mechanical characteristics of concrete and steel have been introduced into the program through characteristic curves in compression and in tension respectively.

The characteristic curves used in numerical modeling are obtained based on the current standards [10, 12] and are displayed in Figure 10.

The nonlinear behavior of concrete had the following mechanical characteristics: compressive strength equal to 12 MPa, the tensile strength equal to 1.6 MPa, the elastic modulus had the value of 27 GPa and the Poisson's coefficient was equal to 0.2. The reinforcement had the yielding limit of 355 MPa and 235 MPa respectively, the elastic modulus of 210 GPa and Poisson's coefficient with the value of 0.3. The introduction in the model of the uniaxial compressive curve of concrete was performed using the Multi-Linear Isotropic Hardening (MISO) option of the computation program [13].

#### 3.4. The finite elements (FE) model and the computation assumptions

The concrete modeling was performed using *quadrilateral* type surface elements and had a nonlinear behavior, associated with cracking in tension and crushing in compression, defined according to the curves given in Figure 10. The reinforcement was modeled with *bar* type elements with the option to take over only axial stress. In the bars modeling the discrete representation mode was used. The reinforcement was connected to the network nodes of the concrete. A perfect bonding between the two materials (concrete and steel reinforcement) was considered in the numerical modelling.

The force applied on the computation model, Figure 11, during the nonlinear analysis was divided in a series of loading steps and sub-steps. The stiffness matrix of the model was automatically adjusted based on the Newton-Raphson equilibrium iterations at the end of each loading step.

The stresses in the first loading step resulted from the self-weight of the element, Figure 12a. The transverse cross-sections from the right-hand side of the stress maps are in the fixed area. The element cracked in the tensioned area and the distribution of the normal stresses is close to a linear one.

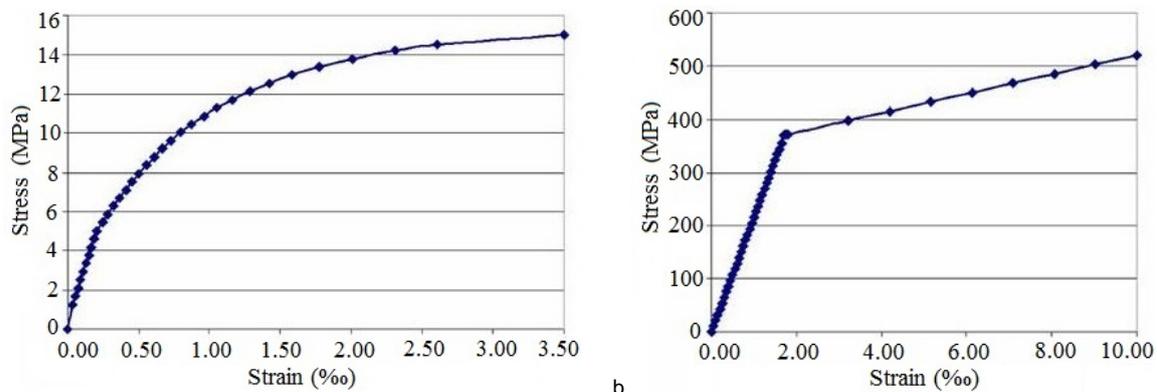


Fig. 10 - Characteristic curves of the materials for numerical modeling / *Curbele caracteristice ale materialelor pentru modelare numerică*: a. characteristic curve of concrete in compression / *curba caracteristică a betonului la compresiune*; b. characteristic curve of steel in tension / *curba caracteristică a oțelului la întindere*.

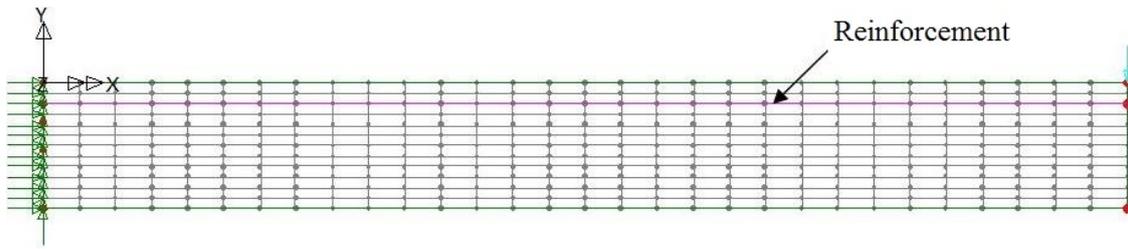


Fig. 11 - Computation model / Model de calcul

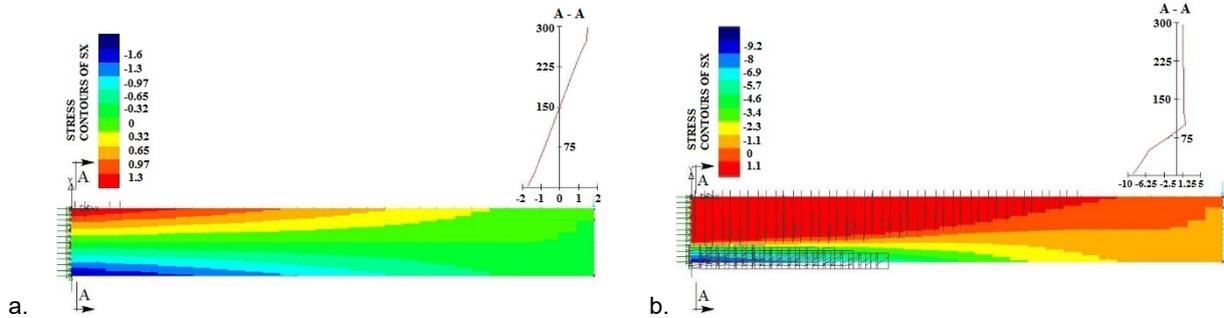


Fig. 12 - a. The normal stresses map for the first loading step / Harta de tensiuni normale pentru prima treaptă de încărcare [MPa]; b. The normal stresses map for an intermediate loading step / Harta de tensiuni normale pentru o treaptă intermediară de încărcare [MPa].

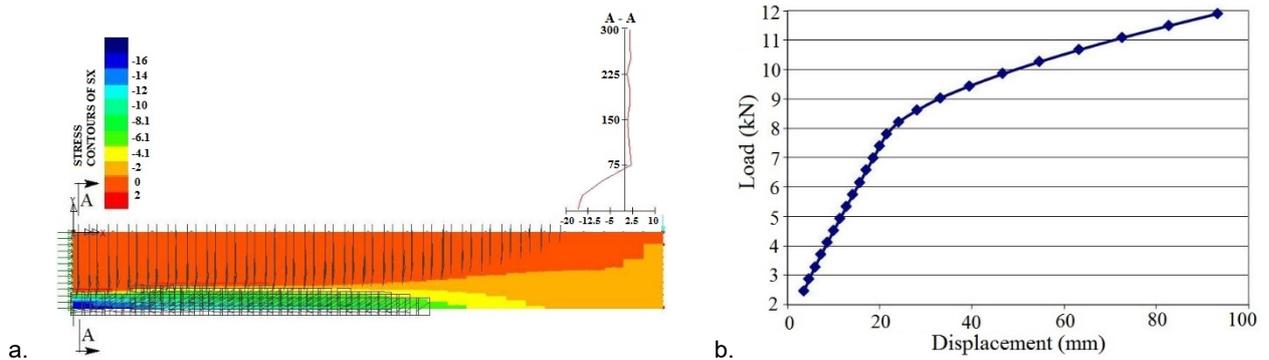


Fig. 13 - a. The normal stresses map for the last loading step / Harta de tensiuni normale pentru ultima treaptă de încărcare [MPa]; b. Load-displacement curve / Curba încărcare-deplasare.

In the intermediate loading step, Figure 12b, the concrete in the tensioned area continued to crack but cracks developed in the compressed area as well. The force for this loading step was considered equal to 6 kN.

In the last loading step, Figure 13 a, the failure of the element is produced through the breaking of the reinforcement. The concrete in the compressed area did not reach the ultimate strain. The force for this loading step was equal to 11.9kN.

The displacement in the force direction at the moment of failure was of 0.093m. The load-displacement curve of the element is presented in Figure 13b.

At the base of the concrete failure mode was the model suggested by William and Warke, and for the reinforcement failure the Von Misses criteria was used.

The shape of the load-displacement curve in Figure 13b is similar to the one displayed in Figure 7 (for the G1 and G2 beams). The finite element analysis has accurately estimated the structural

behavior of the beams and validated the results obtained in the experimental program.

#### 4. Conclusions

The evaluation of the damage level for the reinforced concrete cantilevers underlined that, due to the reduced strength of concrete in tension, the concrete cracks even only under its own weight, obtaining a damage level of the tested elements from the beginning.

Through the experimental testing the following aspects have been confirmed:

- The vibration frequency is a significant guide on the damage level of a reinforced concrete element; the higher the damage level the lower the frequency.
- There is an almost linear relation between frequency and damage level of the element up to the moment in which the concrete cracks heavily in the tensioned area. The frequency remains constant

from the cracking of concrete up to the yielding of reinforcement. After the beginning of reinforcement yielding, the linear relation between frequency and damage level has a lower slope.

- The frequency change of a reinforced concrete element is a very important indicator for the appearance of the first cracks.
- The critical damping fraction increases with respect to the damage level.

The numerical modeling results provided a good confirmation the experimental data.

A permanent monitoring of the structures, in terms of detecting the incipient phase of the dynamic parameters modification, can lead to important benefits regarding the degradation prevention, their evaluation after an earthquake, warning in case of exceeding safety parameters and expansion of the service life.

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