

# MULTIPLE LINEAR REGRESSION MODEL TO PREDICT MECHANICAL PROPERTIES AND IMPACT RESISTANCE OF HOOKED-END STEEL FIBRE-REINFORCED BLENDED CONCRETE

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**Concrete, being one of the most commonly used building materials in construction industries, has, cement as its principle component. Cement paste is responsible for bonding and strength gain in concrete. The production of cement releases an equal amount of CO<sub>2</sub> to the atmosphere causing environmental pollution. This Study identifies the property enhancement of concrete due to the partial replacement of cement with pozzolanic materials such as fly ash and silica fume at 40% and 7% respectively. Tests were conducted to identify mechanical properties – compression, split tension, flexure; and Impact resistance. Steel fibres were also incorporated at 0.75%, 1.15% and 1.55% to the mix. Addition of steel fibre to blended concrete showed an increase of 33-77% on mechanical properties and 186% on impact resistance of concrete at 28 days. A Multiple linear regression model was formulated using SPSS, and consequently, equations were derived to predict the mechanical properties and impact resistance of concrete. The equations were found to be in a good agreement with the experimental results obtained by various other researches with significance level lesser than 0.05 in ANOVA.**

**Keywords:** fly ash; silica fume; steel fibres; mechanical properties; impact resistance; regression

## 1. Introduction

Concrete production has developed rapidly and many researchers are trying to improve the physical and mechanical properties of concrete [1]. Cement is the principle component in the concrete production industry. The production of Portland cement approximately reaches four billion tonnes a year. The key role of Cement is to bind the aggregates and make the concrete components react chemically. Though cement has so many advantageous characteristics, it has become extremely costly and also poses a greater threat to the harmony of the environment [2]. This paved the way for the utilization of certain materials having cementitious properties (commonly called supplementary cementitious materials or SCMs) like fly ash, silica fume, metakaolin, ground granulated blast furnace slag and so on in concrete for sustainable development and improvement in durability aspects [3]. The utilization of these materials has also become extremely important because of their performance in safeguarding the environmental harmony in all aspects in terms of ecological, behavioural and economical methods of disposal of hazardous waste materials, providing a cleaner environment with reduced energy consumption and better strength characteristics [4]. The inclusion of these materials significantly improves the properties of concrete in both fresh and hardened states. These natural pozzolans improve the durability of the concrete by altering the

microstructure and the interfacial transition zone between aggregate-paste or paste-reinforcement bonding [5].

Fly ash has been used as a mineral admixture to partially replace cement or sand [6]. Being smaller sized particle than cement, its major constituent of fly ash is silica (SiO<sub>2</sub>); however, the sum of silica (SiO<sub>2</sub>), alumina (Al<sub>2</sub>O<sub>3</sub>) and Iron oxide (Fe<sub>2</sub>O<sub>3</sub>) contributed to the typical value equal to that in cement. When fly ash is mixed with cement, it makes the cement paste smoother and allows better bonding between aggregate and cement particles, which, in turn, makes the concrete more durable and impervious in nature. Chemically, fly ash undergoes pozzolanic activity where it reacts with CH (calcium hydroxide) and produces secondary C-S-H (Calcium-silicate-hydrate) gel which is similar to that produced in the cement but at a different rate. In concrete, fly ash boosts up the workability without adding extra water by their hard and round shaped particle size. Fly ash reduces pores in concrete during hydration and subsequently makes the concrete a hard and durable one. Fly ash negatively affects the setting time of cement by delaying it. Concrete with fly ash as a replacement to cement (partially) showed a delay in the development of early strength and a better later strength [7,8].

Silica fume is also a by-product obtained in the manufacture of silicon or different forms of silicon alloys. Its major composition is SiO<sub>2</sub> (80-85%); this enabled its use in construction

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industries. Silica fume is a very active mineral admixture in producing high-strength concrete due to its high pozzolanic reactivity [9, 10]. Silica fume in concrete is responsible for better cohesiveness, reduction in segregation and bleeding. Similar to fly ash, silica fume increases the cement paste-aggregate grain bond and decreases the interfacial transition zone porosity; thereby, altering the mechanical properties of concrete [11].

The combined addition of fly ash and silica fume in concrete at optimum amounts not only improves the quality of concrete but also reduces its cost of production. Silica fume can increase the compressive strength of concrete and fly ash reduces the cost of materials without disturbing the environment [12]. This led to the development of high-strength concrete. The use of high-strength has become very popular in the construction industry since 2003. The major drawback of these types of concrete is brittle failure resulting from their low tensile strength and strain. This behaviour makes it not suitable for structures subjected to sudden loads such as, earthquake and impact loads. This can be overcome with the help of the fibre addition [13]. Fibre inhibits crack initiation and prevents its propagation [14]. Addition of fibres to concrete helps in the betterment of structural characteristics of concrete by enhancing compression, tension, flexure and impact resistance. In particular, steel fibres were highly employed for flooring and precast works due to their long-term strength development and toughness resistance for concrete. Steel fibre reinforced concrete is a man-made concrete base composite material [15]. The addition of blended materials like fly ash (as cement replacement) to steel fibres, increases the workability of the concrete mix [16]. ACI 544. 2R [17] also recommends partial replacement of cement with fly ash and silica fume in fibre reinforced concrete can improve mechanical properties of concrete either in fresh or hardened states. Pozzolan addition to fibre concrete reduces the cement content compared to conventional fibre concrete.

This paper identifies the mechanical properties and impact resistance of hooked-end

steel fibre reinforced concrete blended with fly ash and silica fume at 40% and 7% respectively. A regression model was also developed to predict the results obtained from the experiments.

## 2 Experimental Program

### 2.1. Materials

Commercially available ordinary portland cement of grade 53 confirming to BIS 1987 [18] with specific gravity 3.15 was used. The initial and final setting times of cement were observed to be 31 (min) and 315 (min) respectively. The two supplementary cementitious materials added were fly ash of class C category obtained from Neyveli Lignite Corporation, India with a specific gravity 2.18 and Silica fume obtained from Elkem materials conforming to ASTM C1240 [19] with a specific gravity of 2.02. Table 1 shows the chemical composition of cement, fly ash (class C) and silica fume. Locally available dry and clean, natural river sand of specific gravity 2.45 and of grading zone II (from table 4 of BIS 1963 [20]) confirming to BIS 1970 [21] was used as fine aggregate. The crushed coarse aggregate of a nominal size corresponding to 10-12.5 mm with specific gravity 2.65 confirming to BIS 1970 [21] was used as gravel. Steel fibres with both ends hooked, of length 30 mm and diameter 0.5 mm with a corresponding aspect ratio of 60 was also employed in the mix. Master Glenium SKY 8233 (formerly known as Glenium B233), a high-performance super-plasticizer based on polycarboxylic ether (PCE) from BASF India Limited, India, confirming to ASTM C494 [22] was used as a water reducing agent in concrete. This is free of chloride, containing low alkali and is compatible with all types of cement. The above-mentioned admixtures both mineral (fly ash and silica fume) and chemical admixture (Glenium B233) were not processed and used as received. Locally available potable water confirming to BIS 2000 [23] was used for mixing. Figure 1 shows the physical identification of fly ash, silica fume and steel fibres used in the concrete mix.

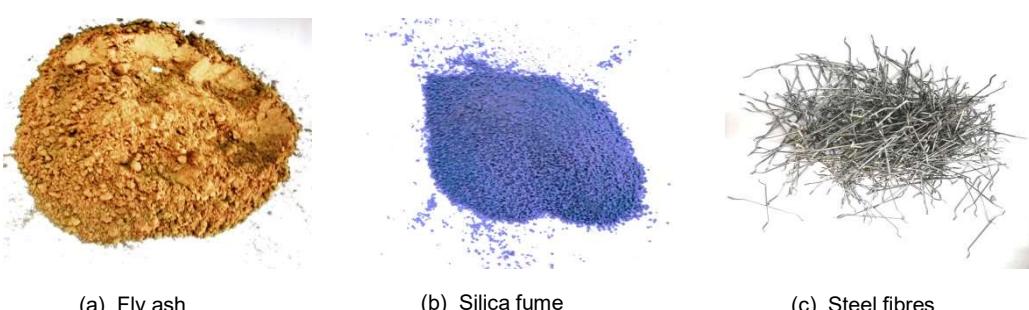


Fig. 1 - Diagrammatic representation of fly ash (a), silica fume (b) and steel fibres (c) used in the concrete mix.

**Table 1**

Material Constituents	Chemical compositions of cement, fly ash and silica fume			
	From Test	Cement (%)	Fly ash- Class C category (%)	Silica fume (%)
Silica (as SiO <sub>2</sub> )	20.2	57.60	-	85.72
Calcium Oxide (lime content) as CaO	63.41	11.64	-	-
Alumina (as Al <sub>2</sub> O <sub>3</sub> )	1.07	15.34	0.06	-
Iron Oxide (as Fe <sub>2</sub> O <sub>3</sub> )	1.07	6.1	0.45	-
Magnesia (as MgO)	1.12	0.40	-	-
Sulphuric anhydride (as SO <sub>3</sub> ), Max	2.02	1.79	-	-
Total loss on ignition, Max	1.48	2.86	1.96	-
Total Chlorides (as Cl)	0.006	0.02	-	-
Sodium Oxide (as Na <sub>2</sub> O)	0.35	0.44	-	-
Potassium Oxide (as K <sub>2</sub> O)	0.95	0.04	-	-
Total alkalis (as Na <sub>2</sub> O)	-	0.47	-	-
SiO <sub>2</sub> +Al <sub>2</sub> O <sub>3</sub> +Fe <sub>2</sub> O <sub>3</sub> (% by mass, min)	-	79.04	-	-

**Table 2**

Mixture ID	Mixture ID of Samples			
	Cement (%)	Fly ash (%)	Silica fume (%)	Steel fibres (%)
M000	100	0	0	0
M001	100	0	0	0.75
M002	100	0	0	1.15
M003	100	0	0	1.55
F400	60	40	0	0.00
F401	60	40	0	0.75
F402	60	40	0	1.15
F403	60	40	0	1.55
FS470	53	40	7	0.00
FS471	53	40	7	0.75
FS472	53	40	7	1.15
FS473	53	40	7	1.55

**Table 3**

Mixture ID	Mixture Proportions						
	Cement (kg/m <sup>3</sup> )	Fly ash (kg/m <sup>3</sup> )	Silica fume (kg/m <sup>3</sup> )	Steel fibres (%)	Water (kg/m <sup>3</sup> )	Fine aggregate (kg/m <sup>3</sup> )	Coarse aggregate (kg/m <sup>3</sup> )
M000	320.0	-	-	-	140	938	1146
M001	320.0	-	-	0.75	140	938	1146
M002	320.0	-	-	1.15	140	938	1146
M003	320.0	-	-	1.55	140	938	1146
F400	192.0	128	-	-	140	938	1146
F401	192.0	128	-	0.75	140	938	1146
F402	192.0	128	-	1.15	140	938	1146
F403	192.0	128	-	1.55	140	938	1146
FS470	169.6	128	22.4	-	140	938	1146
FS471	169.6	128	22.4	0.75	140	938	1146
FS472	169.6	128	22.4	1.15	140	938	1146
FS473	169.6	128	22.4	1.55	140	938	1146

## 2.2 Design Mixture proportions

The mixture proportions were designed in such a way that they influence the behaviour of concrete towards mechanical properties and impact resistance. The mix design was formulated for concrete with a proportion of 1:2.93:3.58 with water-cement ratio as 0.45. The different combinations of the mix were formulated based on the addition of steel fibres at 0%, 0.75%, 1.15% and 1.55%; fly ash and silica fume were added as partial replacements to cement at 40% and 7% respectively. Control mixture consists of only plain cement at 320 kg/m<sup>3</sup>. When fly ash was used for 128 kg/m<sup>3</sup> (40%) the cement content was reduced to 192 kg/m<sup>3</sup>. Addition of silica fume 22.4 kg/m<sup>3</sup> (7%), in turn, reduced the cement content to 169.6

kg/m<sup>3</sup>. Superplasticizer was added as a water reducing admixture to a maximum of 1% as per the requirement of the mix. In addition to control concrete, there were 12 mixes tested and their corresponding proportions of fly ash, silica fume and steel fibres are represented in Table 2 and Table 3.

## 2.3 Mixing Procedure

The fibre reinforced concrete was mixed on the following procedure; Firstly, aggregates (both fine and coarse) were placed and mixed under dry condition; cement and fibres were spread above the aggregates and were dry mixed; Secondly, mixing water (almost 90%) was added and mixed with it; The remaining 10% water and plasticizer

were mixed and spread over the concrete mix. As per BIS 2000, there should be a minimum of 2 minutes as mixing time in concrete. At last, fibre reinforced concrete specimens were cast into specimen moulds and vibrated to remove the air entrapped and simultaneously compacted in three layers. The specimens were set for 24 ± 8 hrs in the laboratory. The specimens after demoulding were immersed in the curing tank under normal curing method for 28 days [16, 24].

### 3. Testing methods

Mechanical properties of concrete were identified by measuring the strength at compression, split tension and flexure. Compression tests were carried out on concrete cubes of size 100 X 100 X 100 (mm) under the standard compression testing machine as per BIS 1959 [25]. For split tension tests, concrete cylinder of 100 mm diameter and 200 mm height were used and the test was performed as per ASTM C496/C496M-11 [26]. Flexure strength was measured using standard prisms of size 500 X 100 X 100 (mm) under symmetrical two-point loading set up as per ASTM C78/C78M-10 [27]. The impact resistance of concrete was measured using the modified impact test on concrete cubes of size 100 X 100 X 100 (mm) as per ACI 1989 [17]. A hammer of weight 135 N was allowed to drop from a height of 413 mm. Spherical steel ball of 64.5 mm diameter was used to transfer the impact load onto the specimen. The Impact resistance was identified based on the energy (Number of blows to achieve failure) it can withstand.

Impact Energy,  $U = (n \times m \times v^2)/2$       (1)  
where, n is number of blows (impact) that the concrete resists

$m = w/g$ ;

m - mass of the hammer;

w – weight of the hammer;  
g - acceleration due to gravity;  
Height of fall =  $g \times t^2/2$ ;  
t - time of drop;  
 $v = g \times t$ ;  
v - velocity of drop.

### 4 Results and discussion

The failure pattern and strength (energy) of concrete under compression, split tension, flexure and impact with their relative strength (in terms of percentage of the strength of control concrete) is represented in Figure 2 and Table 4 respectively.

#### 4.1 Compressive Strength

The compressive strength plot is given in Figure 3. The strength of the normal concrete control specimen (without fly ash, silica fume and steel fibres) was identified to be 36 MPa. Addition of steel fibres at 0.75%, 1.15% and 1.55% to control concrete, increased the strength of control concrete to 4%, 11% and 20% respectively. This is due to steel fibres, which, act as crack arrestors through their bridging mechanism [28]. Steel fibres also undergo pull out process and consequently delays fracture formation and limit its propagation [29].

In comparison to the strength of control concrete, concrete blended with 40% fly ash (as a replacement to cement partially) showed a decrease in strength of 15%, 7%, 2% for 0%, 0.75%, 1.15% steel fibre content respectively and an increase in strength of 2% for 1.55% steel fibre addition. Concrete blended with both fly ash and silica fume showed an increase in strength of 1.4% at the end of 28 days. One of the most undesirable drawbacks of fly ash partial addition to concrete is their delayed early strength; this can be overcome by silica fume. The combined addition of fly ash



(a) Compression



(b) Split tension



(c) Flexure



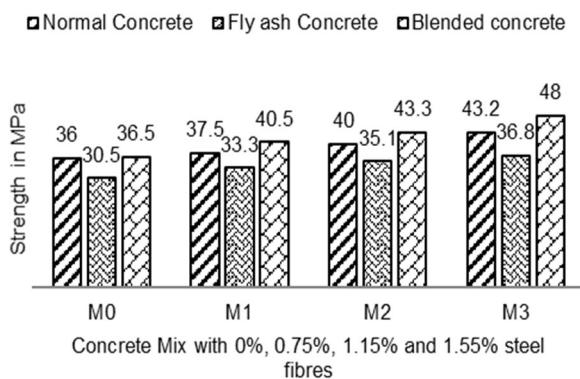
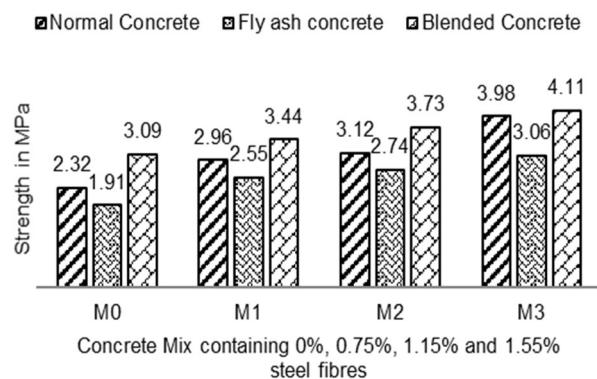
(d) Impact

Fig. 2 - Failure of concrete in compression (a), split tension (b), flexure (c) and impact (d).

**Table 4**

Mixture ID.	Mechanical Properties and Impact Resistance of fibre reinforced concrete										
	Fly ash (%)	Silica fume (%)	Steel fibres (%)	$f_c$ (MPa)	Relative $f_c$ (%)	$f_{st}$ (MPa)	Relative $f_{st}$ (%)	$f_f$ (MPa)	Relative $f_f$ (%)	$I_E$ (kN mm)	Relative $I_E$ (%)
M000	-	-	-	36.0	100	2.32	100	4.2	100	780.57	100
M001	-	-	0.75	37.5	104	2.96	128	5.4	129	1505.38	193
M002	-	-	1.15	40.0	111	3.12	134	6.2	148	1895.66	243
M003	-	-	1.55	43.2	120	3.98	172	6.6	157	2564.72	329
F400	40	-	-	30.5	85	1.91	82	3.4	81	446.04	57
F401	40	-	0.75	33.3	93	2.55	110	3.8	90	1059.34	136
F402	40	-	1.15	35.1	98	2.74	118	5.0	119	1282.36	164
F403	40	-	1.55	36.8	102	3.06	132	5.8	138	1672.64	214
FS470	40	7	-	36.5	101	3.09	133	4.4	105	613.30	79
FS471	40	7	0.75	40.5	113	3.44	148	5.6	133	1616.89	207
FS472	40	7	1.15	43.3	120	3.73	161	6.4	152	1951.41	250
FS473	40	7	1.55	48.0	133	4.11	177	6.8	162	2230.19	286

\* $f_c$  – compressive strength;  $f_{st}$  – split tensile strength;  $f_f$  – flexural strength;  $I_E$  – final impact energy

**Compressive Strength Test results at 28 days****Fig. 3 - Compressive Strength at 28 days.****Split Tensile Strength Test results at 28 days****Fig. 4 - Split Tensile Strength at 28 days.**

and silica fume contribute to the development of strength both in early and later ages [30]. Addition of steel fibres to blended concrete (40% fly ash and 7% silica fume) at 0.75%, 1.15% and 1.55% showed an increase of 12.5%, 20.28% and 33.33% respectively in strength than control concrete. The combination of supplementary cementitious materials to steel fibre reinforced concrete affects the workability of concrete negatively i.e. it decreases the workability. Though silica fume causes brittleness to concrete, their combined addition with steel fibres improves workability [31].

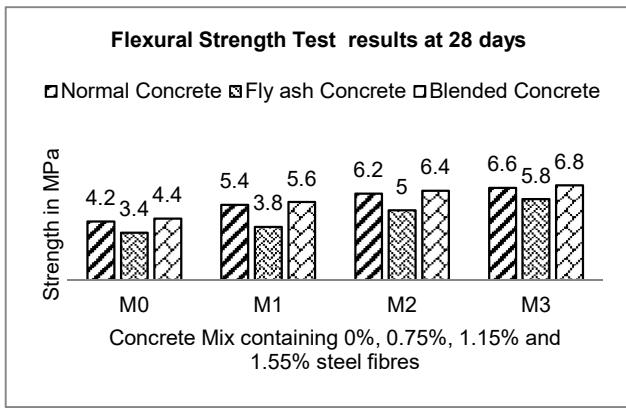
#### 4.2 Split Tensile Strength

Figure 4 represents the plot for strength of concrete in split tension. The split tensile strength for control specimen (without fly ash, silica fume and steel fibres) was found to be 2.32 MPa. With

0.75%, 1.15% and 1.55% steel fibre addition, there has been 28%, 35% and 72% increase in strength respectively in comparison to control concrete. Fly ash concrete (40% fly ash replacement for cement) showed a decrease of 18% for 0% and an increase of 10%, 18%, 32% for 0.75%, 1.15% and 1.55% steel fibre addition respectively. The blended concrete (combination of 40% fly ash and 7% Silica fume as cement replacements) showed an increase in strength of 33%, 48%, 60% and 77% for 0%, 0.75%, 1.15% and 1.55% steel fibre addition respectively.

#### 4.3 Flexural Strength

The strength of concrete in flexure for all the mixes is shown in Figure 5. The flexural strength of control concrete was found to be 4.2 MPa. At 0.75%, 1.15% and 1.55% steel fibre

**Fig. 5** - Flexural strength at 28 days

addition, the strength of concrete had an increment of 29%, 47% and 57% respectively as compared to control concrete. When fly ash was used as a replacement to cement partially, a decrease in strength of nearly 20%, 10% for 0% and 0.75% and an increase in strength of 20%, 38% for 1.15% and 1.55% steel fibre addition respectively in comparison to the control concrete. The combination of fly ash (40%) and silica fume (7%) replacement for cement showed an increase in strength of 4%, 33%, 52% and 62% for 0%, 0.75%, 1.15% and 1.55% steel fibre addition respectively.

#### 4.4 Impact Resistance

From equation (1), the impact energy of concrete is derived as follows,

$$\text{Mass of hammer} = w/g = 135/9810;$$

$$m = 0.0138 \text{ N}$$

$$\text{Height of fall} = g \times t^2/2;$$

$$413 = 9810 \times t^2/2;$$

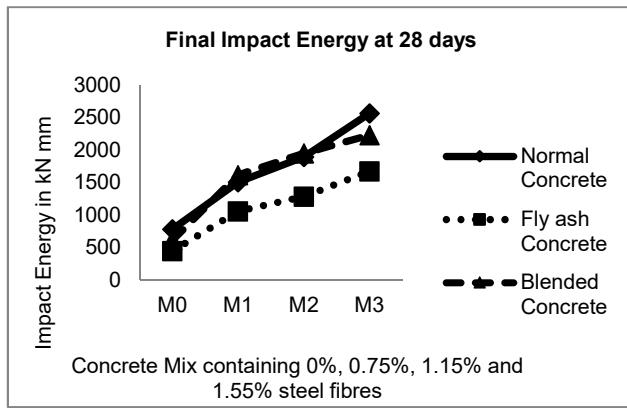
$$t = 0.29 \text{ s};$$

$$v = 2846.58 \text{ mm/s};$$

$$\text{For 1 blow, Impact Energy} = 55.755 \text{ kN mm.}$$

The plot for final Impact energy is given in Figure 6.

The final Impact energy of control concrete was observed to be 780.57 kN mm (14 blows). For 0.75%, 1.15% and 1.55% steel fibre addition, an increase of 93%, 143% and 229% respectively in final impact energy of concrete is observed. Steel fibres drastically improve the impact resistance of concrete. When fly ash was used to replace cement partially at 40%, the strength was decreased to about 43% for 0% and increased to 36%, 64% and 114% for 0.75%, 1.15% and 1.55% steel fibre addition respectively in comparison with the control concrete. The combined addition of fly ash and silica fume showed decrease of 21% for 0% and increase of 107%, 150% and 186% for 0.75%, 1.15% and 1.55% addition of steel fibres respectively. The blended concrete and normal concrete behave in a similar manner in providing impact resistance to concrete with slight increments in final impact energy at the end of 28 days.

**Fig. 6** Impact resistance at 28 days

## 5 Statistical Interpretation of test results

Regression Analysis, a statistical technique was used to estimate the cause-effect relationship among variables. The basic idea of multiple regression analysis is to relate a dependent variable with two or more independent variables.

In particular, a multiple linear regression model is a statistical model which relates a dependent variable with more than one independent variable. This test also helps in predicting the dependent variable from two or more independent variables accordingly. The formula for prediction using regression analysis [32, 33] is given below,

$$y = \beta_0 + \beta_1 \cdot x_1 + \beta_2 \cdot x_2 + \dots + \beta_n \cdot x_n + \epsilon \quad (2)$$

y = Dependent variable

$x_i$  = Independent variable

$\beta_i$  = Parameter

$\epsilon$  = Error

The cause-effect relationship was identified between the percentages of fly ash, silica fume and steel fibres to predict strength in compression ( $f_c$ ), split tension ( $f_{st}$ ), flexure ( $f_f$ ) and impact resistance ( $I_E$  in terms of energy) using multiple linear regression model at the end of 28 days.

Addition of fly ash, silica fume and steel fibres for concrete have a greater effect in property enhancement and long term strength. A working hypothesis can be derived from the findings of Yu et al. 2017 [34], which states, "Addition of supplementary cementitious materials like fly ash and silica fume as partial replacement to cement in concrete can improve the mechanical properties of concrete". From this, a research hypothesis have been formulated from the observed experimental results as, "Addition of fly ash (40%), silica fume (7%) and steel fibres (0.75%, 1.15% and 1.55%) has a positive influence on improving the mechanical properties and impact resistance of concrete".

For this, the dependent variable (predictors) will be the strength in compression ( $f_c$ ), split tension ( $f_{st}$ ), flexure ( $f_f$ ) and impact resistance

**Table 5**

ANOVA					
Model	Sum of Squares	Df	Mean Square	F	Sig.
Regression	730.879	3	243.626	193.409	0.000
Residual	40.309	32	1.260		
Total	771.188	35			

\*Df-Degree of freedom; F-F statistic value; sig.-significance level.

**Table 6**

Model Summary				
Model	R	R Square	Adjusted R Square	Standard Error of the Estimate
1	0.974	0.948	0.943	1.12234

\*R-correlation coefficient; R square- Regression coefficient.

( $I_E$  in terms of energy) and independent variables will be percentages of fly ash, silica fume and steel fibres

### 5.1 Equations for prediction of Strength and Energy

#### 5.1.1 ANOVA

Analysis of Variance (ANOVA) helps in predicting the relationship between the variables. Table 5 shows the results obtained for ANOVA during regression analysis between compressive strength and percentage addition of fly ash, silica fume, steel fibres. Here, significant value is obtained to be 0.000 (lesser than 0.05) indicating that fly ash, silica fume and steel fibre have greater influence in affecting the compressive strength of concrete [35,36].

From Table 6, R is observed to be greater than 0.5, indicating a strong correlation between the dependent and independent variables. Moreover, the R square value (0.948), signifies 94.8% of the variance of the dependent variable can be explained by the regression equation. This proves that the model predicts the dependent variable very well. This implies that the proposed research hypothesis agrees well with the experimental results.

#### 5.1.2 Compressive Strength

Yazici, Inan, and Tabak 2007 [37], proposed equation for predicting strengths in compression, split tension and flexure in terms of fibre properties (aspect ratio ( $l/d$ ) and percentage of steel fibre added ( $V_f$ )). In a similar manner, equation is predicted for compressive strength in terms of percentage of fly ash and silica fume replaced and steel fibre added is proposed as follows,

$$f_c \text{ (MPa)} = 34.772 - 0.131(\% \text{ of Fly ash}) + 1.164(\% \text{ of Silica fume}) + 5.220 (\% \text{ of steel fibre}) + \epsilon, R^2 = 0.948 \quad (3)$$

$\epsilon$  represents an error in the analysis. As the analysis is found to be highly significant, the error is very minimum and hence can be neglected. From experimental observation, the strength of the control specimen (without fly ash, silica fume and steel fibre) at 28 days was found to be 36 MPa. The constant parameter ( $\beta_0$  - 34.772) in the

regression analysis can be represented in terms of percentage of the strength of control concrete. Thus, the equation is proposed with minor modifications as follows;

$$f_c \text{ (MPa)} = [\beta_{c0}] + [\beta_{c1} \cdot x_1] + [\beta_{c2} \cdot x_2] + [\beta_{c3} \cdot x_3], R^2 = 0.948 \quad (4)$$

$$\beta_{c0} = 96.59\% f_{c-c}; \beta_{c1} = - 0.131; \beta_{c2} = 1.164; \beta_{c3} = 5.220;$$

where,  $f_c$  = Predicted compressive strength for respective addition of fly ash, silica fume and steel fibres at 28 days;  $f_{c-c}$  = strength of control concrete in compression at 28 days.  $\beta_{c0}$ ,  $\beta_{c1}$ ,  $\beta_{c2}$  and  $\beta_{c3}$  represent regression parameters for compression in the analysis;  $x_1$ ,  $x_2$ ,  $x_3$  represent the percentage replacement of fly ash, silica fume and addition of steel fibre respectively.  $x_1$ ,  $x_2$ ,  $x_3$  is common for predicting all dependent variables (split tensile strength, flexure strength and final impact energy).

In a similar manner, the equations were predicted for other dependent variables; split tensile strength, flexure strength, impact energy.

#### 5.1.3 Split Tensile strength

$$f_{st} \text{ (MPa)} = [\beta_{t0}] + [\beta_{t1} \cdot x_1] + [\beta_{t2} \cdot x_2] + [\beta_{t3} \cdot x_3], R^2 = 0.948 \quad (5)$$

$$\beta_{t0} = 105.26\% f_{st-c}; \beta_{t1} = - 0.013; \beta_{t2} = 0.146; \beta_{t3} = 0.786.$$

where,  $f_{st}$  is the predicted value of split tensile strength of concrete containing respective additions of fly ash, silica fume and steel fibres at 28 days;  $f_{st-c}$  represents strength of control concrete in split tension at 28 days.

#### 5.1.4 Flexure strength

$$f_f \text{ (MPa)} = [\beta_{f0}] + [\beta_{f1} \cdot x_1] + [\beta_{f2} \cdot x_2] + [\beta_{f3} \cdot x_3], R^2 = 0.963 \quad (6)$$

$$\beta_{f0} = 102.05\% f_{f-c}; \beta_{f1} = - 0.027; \beta_{f2} = 0.186; \beta_{f3} = 1.582.$$

where,  $f_f$  is the predicted value of flexure strength of concrete containing respective additions of fly ash, silica fume and steel fibres at the end of 28 days;  $f_{f-c}$  represents strength of control concrete in flexure at 28 days.

#### 5.1.5 Impact Energy

$$I_E \text{ (kN mm)} = [\beta_{i0}] + [\beta_{i1} \cdot x_1] + [\beta_{i2} \cdot x_2] + [\beta_{i3} \cdot x_3], R^2 = 0.962 \quad (7)$$

**Table 7**

Comparison of compressive strength values from Nochaiya, Wongkeo, and Chaipanich 2010 [38] with the predicted values from the proposed equation (4)

Mix	Fly ash (%)	Silica fume (%)	Nochaiya, Wongkeo, and Chaipanich 2010 [38] results (MPa)	Predicted results from the proposed equation (4) (MPa)	Difference	% Error
PC	-	-	43.20	41.73	+1.47	+3.40
5FA	5	-	42.90	41.07	+1.83	+4.27
5FA2.5SF	5	2.5	47.20	43.98	+3.22	+6.82
5FA5SF	5	5	48.20	46.89	+1.31	+2.72
10FA	10	-	41.50	40.42	+1.08	+2.60
10FA2.5SF	10	2.5	45.50	43.33	+2.17	+4.77
10FA5SF	10	5	46.20	46.24	-0.04	-0.09
20FA	20	-	37.50	39.11	-1.61	-4.29
20FA5SF	20	5	42.00	44.93	-2.93	-6.98
20FA10SF	20	10	43.00	50.75	-7.75	-18.02
30FA	30	-	33.50	37.8	-4.30	-12.84
30FA5SF	30	5	35.80	43.62	-7.82	-21.84
30FA10SF	30	10	36.50	49.44	-12.94	-35.45

\*PC-plain concrete (Control); FA-Fly ash; SF-Silica fume.

**Table 8**

Comparison of compression and split tension results from experimental tests Nili and Afroughsabet 2010 [39] with predicted values from the proposed equations (5) and (6)

Mix	Silica fume (%)	Steel fibre (%)	Compression			Split Tension		
			Experimental results from Nili and Afroughsabet 2010 [39], in MPa	Predicted results from the proposed equation (5) in MPa	% Error	Experimental results from Nili and Afroughsabet 2010 [39] in MPa	Predicted results from the proposed equation (6) in MPa	% Error
1	-	-	41.30	39.89	+3.41	3.22	3.39	-5.28
2	-	0.5	46.35	42.5	+8.31	3.84	3.78	+1.56
3	-	1	47.25	45.11	+4.53	5.22	4.18	+19.92
4	8	-	49.88	49.2	+1.36	3.52	4.56	-29.55
5	8	0.5	53.79	51.81	+3.68	4.26	4.95	-16.20
6	8	1	55.30	54.42	+1.59	5.59	5.34	+4.47

$$\beta_{10} = 110.42\% I_{E-c}; \beta_{11} = -14.171; \beta_{12} = 69.030; \beta_{13} = 977.676.$$

where,  $I_E$  represents the predicted value of Impact energy of concrete containing respective additions of fly ash, silica fume and steel fibres at the end of 28 days;  $I_{E-c}$  indicates Impact energy of control concrete specimen at 28 days.

From the equations it can also be identified that fly ash addition causes a decrease strength and energy (negative sign in parameter  $\beta_2$  (for predicting all dependent variables)) whereas silica fume and steel fibres tend to increase the strength and energy of concrete (due to positive signs for parameters  $\beta_3$  and  $\beta_4$ ). The proposed equations (4, 5, 6 and 7) predicted were based on 95% confidence interval where dependent and independent variables were highly correlated to each other with R (correlation coefficient) value greater than 0.5.

## 5.2 Validation of Test results

### 5.2.1 Validation of Compressive Strength

Nochaiya, Wongkeo, and Chaipanich 2010 [38], evaluated the combination of fly ash (class C with CaO content 15.2%) and silica fume as partial replacement to cement in improving the compressive strength of concrete. This is validated

with the proposed equation for compression (4). The difference between observed and predicted values is represented in Table 7.

As the fly ash and silica fume content increases, the percentage of error was also increased; the results showed minimal error till concrete mix corresponding to 30FA beyond which (i.e for 30FA5SF and 30FA10SF) the error is observed to be high. The proposed equation (4) fits well with fly ash content  $\leq 40\%$  and silica fume content  $\leq 7\%$ .

### 5.2.2 Validation of Compressive Strength, Split Tensile Strength

The proposed equations (4 and 5) are used to compare the actual results obtained by Nili and Afroughsabet 2010 [39], by adding 8% silica fume with 0.5% and 1% steel fibre. The compression and split tensile test results are tabulated as shown below (Table 8).

The maximum error is observed to be around 8% in compression and almost 30% in split tension. This implies that the accuracy in predicting compressive strength is higher compared to split tensile strength. The steel fibres fall within the range, i.e.  $\leq 1.55\%$  and there observed a slight increase in silica fume content (an increase of 1%); the errors were in the

**Table 9**

Comparison of Impact Energy results from experimental results of Yildirim, Ekinci, and Findik 2010 [40] with predicted values from the proposed equation (7)

Mix	No. of Impact (assumed from Figure 2 from Yildirim, Ekinci, and Findik 2010 [40])	Calculated Experimental results from Yildirim, Ekinci, and Findik 2010 [40] in kN mm	Predicted results from the proposed equation (7) in kN mm	No. of Impact that the concrete withstands from proposed equation (7)	Difference in results in terms of no. of Impact	Difference in results (in terms of energy)	% Error
Control	14	781.20	862.60	15	-1	-81.4	-10.42
0.5% steel	32	1785.60	1351.44	24	8	434.16	24.31
0.75% steel	34	1897.20	1595.88	29	5	301.32	15.88
1% steel	42	2343.60	1840.28	33	9	503.32	21.48

acceptable range due to changes in physical and chemical properties of cement, silica fume and steel fibres. The proposed equations (4 and 5) are found to fit very well in this case too.

### 5.2.3 Validation for Impact Energy

The impact Energy is validated from Figure 2 of Yildirim, Ekinci, and Findik 2010 [40], the experimental values (No. of blows) were taken approximately and impact energy was calculated using the equation (1) and compared with the predicted results by substituting into the proposed equation (8) and is represented in Table 9;

The experimental results observed by Yildirim, Ekinci, and Findik 2010 [40] comprise of two kinds of fine aggregates (FA1-0 to 0.5 mm sieve span with  $2.62 \text{ g/cm}^3$  density and FA2-0.5 to 4 mm sieve span with  $2.65 \text{ g/cm}^3$  density) and two different coarse aggregates (CA1- 4 to 16 mm sieve span and CA2-16 to 22 mm sieve span with density  $2.7 \text{ g/cm}^3$ ). The error observed may be due to aggregates, grade of cement, variation in water to cement ratio or any other factors. Table 9 implies that the equation predicted is good at estimating the impact energy with minimal error.

From the validations, it can be stated that the equations (4, 5, 6 and 7) predict mechanical properties and impact resistance of concrete mix at the end of 28 days very well.

## 6 Limitations

- The equations proposed are valid for cement (OPC 53 grade), class C fly ash and Silica fume with the prescribed physical and chemical properties.
- Water to cement ratio should be 0.45. (Difference in w/c ratio is also acceptable and checked appropriately if chemical admixtures were added)
- Steel fibres should be of hooked-end type with length 30 mm and aspect ratio 60.
- The accuracy of the results in the prediction using multilinear regression equations depends on the maximum number of inputs (tested values or trials) incorporated in the analysis.

The above-mentioned criteria do not limit the prediction; instead, indicate that the equations

can be used to predict the strength and energy with minimal error under these conditions.

## 7 Conclusion

This study identifies the effect of addition of steel fibres to concrete containing fly ash and silica fume using experimental procedure and statistical investigation. The findings of the research were listed as follows:

- Concrete with 0.75%, 1.15% and 1.55% addition of steel fibres showed a maximum increase of 20% in compressive strength, 72% in split tensile strength, 57% in flexure strength and 93 to 29% in final impact energy
- Fly ash (40% replacement) to concrete had a decrease in strength of 15%, 18%, 19% and 43% in compression, split tension, flexure and final impact energy respectively.
- Addition of steel fibres to fly ash concrete showed an increase in strength to a maximum of 2% in compression, 32% in split tension, 38% in flexure and 114% in final impact energy at the end of 28 days.
- The combination of 40% fly ash and 7% silica fume exhibited better mechanical properties by increasing the strength at 1%, 33% and 5% in compression, split tension and flexure respectively, but exhibited poor performance in improving impact resistance of concrete by reducing the impact energy to 79%. This can be overcome by adding steel fibres.
- Steel fibre addition at 0.75%, 1.15% and 1.55% to blended concrete (40% fly ash and 7% silica fume) had a maximum increase in strength of 33% in compression, 77% in split tension, 62% in flexure and 186% in final impact energy.
- In regression analysis, the proposed research hypothesis stating that the addition of fly ash, silica fume and steel fibres enhanced the mechanical properties and impact resistance of concrete can be accepted. The regression coefficients for all the dependent variable predictions were observed to be between 0.9 and 1 implying that equations (4, 5, 6 and 7) predict the dependent variables very well.
- The significance level of ANOVA analysis was obtained to be less than 0.05, indicating that

the equations are highly significant and can be interpreted that addition of fly ash, silica fume and steel fibre influence the enhancement of strength in compression, split tension, flexure and impact resistance of concrete. The predicted results were found to be in good agreement with the experimental results.

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#### REFERENCES

1. R. Mačiulaitis, M. Vaičienė and R. Žurauskienė, The effect of concrete composition and aggregates properties on performance of concrete, *Journal of Civil Engineering and Management*, 2009, **15**(3), 317.
2. S. Janani and A. S. Santhi, Multiple Linear Regression Model for Mechanical Properties and Impact Resistance of Concrete with Fly Ash and Hooked-end Steel Fibers, *International Journal of Technology*, 2018, **9**(3), 526.
3. P. Duan, C. Yan, and W. Zhou, Compressive strength and microstructure of fly ash based geopolymers blended with silica fume under thermal cycle, *Cement and Concrete Composites*, 2017, **78**, 108.
4. M. Kosior-Kazberuk and M. Lelusz, Strength development of concrete with fly ash addition, *Journal of Civil Engineering and Management*, 2007, **13**, 115.
5. Z. Zhang, B. Zhang and P. Yan, Comparative study of effect of raw and densified silica fume in the paste, mortar and concrete, *Construction and Building Materials*, 2016, **105**, 82.
6. C. H. Lin, F. S. Ling and C. H. Hwang, Flexural behavior of high strength fly ash concrete beams, *Journal of the Chinese Institute of Engineers*, 1992, **15**(1), 85.
7. C. Hu, Microstructure and mechanical properties of fly ash blended cement pastes, *Construction and Building Materials*, 2014, **73**, 618.
8. S. Hnsolu and O. Bayrak, Optimization of early flexural strength of pavement concrete with silica fume and fly ash by the Taguchi method, *Civil Engineering and Environmental Systems*, 2004, **21**(2), 79.
9. J. Yajun and J. H. Cahyadi, Effects of densified silica fume on microstructure and compressive strength of blended cement pastes, *Cement and Concrete Research*, 2003, **33**, 1543.
10. G. Skripkiunas, Z. Rudzionis, V. Vaitkevicius, Complex admixtures for high-strength concrete, *Journal of Civil Engineering and Management*, 2002, **3**(4), 276.
11. P. Dybe and K. Furtak, Influence of silica fume content on the quality of bond conditions in high-performance concrete specimens, *Archives of Civil and Mechanical Engineering*, 2017, **7**, 795.
12. Z. Rong, W. Sun, H. J. Xiao and W. Wang, Effect of silica fume and fly ash on hydration and microstructure evolution of cement based composites at low water – binder ratios, *Construction and Building Materials*, 2014, **51**, 446.
13. H. Y. Leung and R. V. Balandran, Flexural behaviour of concrete beams internally reinforced with GFRP rods and steel rebars, *Structural Survey*, 2003, **21**(4), 146.
14. W. T. Lin, R. Huang, J. J. Chang and C. L. Lee, Effect of silica fumes on the permeability of fiber cement composites, *Journal of the Chinese Institute of Engineers*, 2009, **32**(4), 531.
15. W. Abbass, M. Iqbal Khan and S. Mourad, Evaluation of mechanical properties of steel fiber reinforced concrete with different strengths of concrete, 2018, **168**, 556.
16. C. D. Alis and O. Karahan, Properties of steel fiber reinforced fly ash concrete, *Construction and Building Materials*, 2009, **23**, 392.
17. ACI (American Concrete Institute), 544.2R:1989 Measurement of properties of fibre reinforced concrete
18. BIS (Bureau of Indian Standards), 12269:1987 Ordinary Portland cement 53 grade specification (reaffirmed 2013)
19. ASTM C1240-11. Standard specification for silica fume used in cementitious mixtures.
20. BIS (Bureau of Indian Standards) 2386 (part 3):1963 Methods of tests for aggregates for concrete.
21. BIS (Bureau of Indian Standards) 383:1970 Specifications for coarse and fine aggregates from natural sources for concrete
22. ASTM C494/C494M-11 Standard specification for chemical admixtures for concrete
23. BIS (Bureau of Indian Standards) 456:2000 Code of practice for plain and reinforced concrete (reaffirmed 2007)
24. A. R. Hariharan, A. S. Santhi and G. Mohan Ganesh, Statistical model to predict the mechanical properties of binary and ternary blended concrete using regression analysis, *International Journal of Civil Engineering*, 2015, **13**, 331.
25. BIS (Bureau of Indian Standards) 516:1959 Methods of tests for strength of concrete (reaffirmed 1999)
26. ASTM C496/C496M-11 Standard Test Method for Splitting Tensile Strength of Cylindrical Concrete Specimens
27. ASTM C78/78M-10 Standard test method for flexural strength of concrete.
28. N. Banthia and J. F. Trottier, Concrete reinforced with deformed steel fibers Part II: Toughness characterization, *ACI Materials Journal*, 1995, **92**, 146.
29. N. Kurihara, M. Kunieda, T. Kamada, Y. Uchida and K. Rokugo, Tension softening diagrams and evaluation of properties of steel fiber reinforced concrete, *Engineering Fracture Mechanics*, 2000, **65**, 235.
30. H. Yazici, The effect of silica fume and high volume class C fly ash on mechanical properties, chloride penetration and freeze-thaw resistance of self-compacting concrete, *Construction and Building Materials*, 2008, **22**, 456.
31. F. Koksal, F. Altun, I. Yigit and Y. Sahin, Combined effect of silica fume and steel fiber on the mechanical properties of high strength concretes, *Construction and Building Materials*, 2008, **22**, 1874.
32. M. H. Alhamaydeh, S. A. Barakat and F. H. Abed, Multiple regression modeling of natural rubber seismic-isolation systems with supplemental viscous damping for near-field ground motion, *Journal of Civil Engineering and Management*, 2014, **19**(5), 665
33. J. S. Chen and S. C. Wang, Statistically modelling relationships between accident types and highway features, *Civil Engineering and Environmental Systems*, 1999, **16**(1), 51.
34. J. Yu, C. Lu, C. K. Y. Leung and G. Li, Mechanical properties of green structural concrete with ultrahigh-volume fly ash, *Construction and Building Materials*, 2017, **147**, 510.
35. H. Tanyildizi, Variance analysis of crack characteristics of structural light weight concrete containing silica fume exposed to high temperature, *Construction and Building Materials*, 2013, **47**, 1154.
36. G. C. H. Kostoglouidis, J. Anagnostou, C. H. Ftikos and J. Marinos, Development of a multiple linear regression model for predicting the 28-day compressive strength of portland pozzolan cement, *Materials and Structures*, 1999, **32**, 98.
37. S. Yazici, G. Inan and V. Tabak, Effect of aspect ratio and volume fraction of steel fiber on the mechanical properties of SFRC, *Construction and Building Materials*, 2007, **21**, 1250.
38. T. Nochhaiya, W. Wongkeo and A. Chaipanich, Utilization of fly ash with silica fume and properties of Portland cement - fly ash - silica fume concrete, *Fuel*, 2010, **89**, 768.
39. M. Nili, and V. Afroughsabet, Combined effect of silica fume and steel fibers on the impact resistance and mechanical properties of concrete, *International Journal of Impact Engineering*, 2010, **37**, 879.
40. S.T. Yildirim, C. E. Ekinci and F. Findik, Properties of hybrid fiber reinforced concrete under repeated impact loads, *Russian Journal of Non-destructive Testing*, 2010, **46**, 538.

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