# ANALIZA WEIBULL A REZISTENȚEI LA IMPACT A BETONULUI AUTOCOMPACTANT RANFORSAT CU CFRP RECICLAT
# WEIBULL RELIABILITY ANALYSIS OF IMPACT RESISTANCE ON SELF-COMPACTING CONCRETE REINFORCED WITH RECYCLED CFRP PIECES

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***The variations in impact strength of self-compacting concrete reinforced with recycled Carbon Fibre Reinforced Polymer (CFRP) Pieces were examined using Weibull reliability analysis. For this purpose, the experimental results of an earlier research were investigated statistically using the two parameter Weibull distribution. The shape and scale parameter of Weibull distribution function was determined from five statistical methods namely Least-Squares (LS) regression of Y on X, Least-Squares (LS) regression of X on Y, Empirical Method (EM), Energy Factor Pattern Method (EPM) and Graphical Method (GM). The Weibull parameters were used to describe the impact strengths (number of blows required to cause first crack and failure) in terms of reliability. Further, regression equations were developed between the impact strength and reliability of self-compacting concrete reinforced with recycled CFRP Pieces. In order to validate the developed linear regression equations, six reliable statistical indicators namely Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE), Mean Absolute Deviation (MAD), Integral Absolute Error (IAE), Relative Root Mean Square Error (RRMSE) and Relative Percentage Error (RPE) have been used. Results suggested that the LS- X on Y, EM and EPM are more effective to estimate the Weibull parameters. The developed linear regressions equations which were validated by six statistical indicators show a good fit and higher accuracy.***

***Keywords:*** *Weibull, Impact strength, Least-square, Reliability, Shape parameter, Scale parameter*

## 1. Introduction

It is estimated that every year approximately a sum of five thousand tons of CFRP wastes are buried in the ground, or left as they are in the universe [1]. Over the years the utilisation of recycled CFRP waste as fibres has increased tremendously in construction industry as they exhibit a high strength even at a low density [2, 3]. It is perhaps worthwhile to indicate that the several researches were conducted in laboratories to evaluate the mechanical properties of plain and fibre reinforced concrete (FRC) containing recycle particles [3-10]. However, studies about the impact resistance of recycled CFRP FRC, especially self-compacting concrete are quite limited in numbers [11]. Impact strength are measured by several methods such as charpy test, projectile impact test, explosive test and drop weight test, among them, drop weight is the simplest and attractive method suggested by the ACI committee 544 [12].

The research [13] Reported the variation in impact resistance of steel FRC and plain concrete determined from a drop weight test. Results suggested that a minimum of 36 and 143 specimens per concrete mix were required to assure an error below 10% and 5% respectively. Song et al. 2005 [14] Statistically investigated the strength reliability of steel–polypropylene hybrid FRC and steel FRC subjected to drop weight test. Kolmogorov-Smirnov test and Kaplan–Meier analysis were also performed for normality and reliability test respectively. The Kolmogorov-Smirnov test indicated that the strength of two FRCs hardly followed the normal distributions and the Kaplan-Meier analysis revealed that the hybrid FRC improves the impact strength reliability a little higher than the steel FRC. Song et al. 2005 [15] Examined the variations in impact resistance of high-strength steel FRC, versus those of high-strength concrete (HSC) statistically. The Kolmogorov-Smirnov test indicated that the distribution of first-crack and failure strengths of HSC was approximately normal while the high-strength steel FRC was poorly normally distributed.

The research [16] used the statistical approach for examining the variations in impact strength of polypropylene FRC subjected to drop-weight impact test suggested by ACI Committee 544 [12]. The statistical analysis indicated that in order to assure an error below 10%, at least 40 specimens per concrete mix is required for testing. Chen et al. 2011 [17] investigated the combined effect of steel fibre and steel rebar on the impact resistance of plain concrete and FRC under drop weight test. In the view of variations of experimental test results,

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they were analyzed using two parameter Weibull distribution. The combination of steel fibre and steel rebar allowed much more beneficial positive composite effect on the improvement of impact strength than that of the samples containing only steel fibres or rebars. It was proved that the probabilistic distributions of the number of blows needed to cause first crack and final failure in six types of samples approximately follow two-parameter Weibull distribution. Rahmani et al. 2012 [18] carried out the comprehensive statistical analysis to evaluate the variations in impact resistance of cellulose, polypropylene, and steel FRC. The first crack strength of the steel and cellulose FRC specimens shows significant improvement over the plain concrete, which is confirmed by the p-value equal to zero in the Kruskal-Wallis test. The p-value of the Kruskal–Wallis test for the first crack strength of polypropylene FRC specimens is 0.093. Also, the experimental results showed that the increase in the number of post-first crack blows, polypropylene and steel fibres had a remarkable effect than cellulose fibres. Mastali and Dalvand 2016 [19] carried out experimental and statistical studies to investigate the distribution of mechanical properties of self-compacting concrete reinforced with recycled CFRP pieces. Reported the compressive strength, flexural strength and impact resistance of reinforced self-compacting concrete specimens follow a normal distribution.

However a greater scatter in drop weight test results [13-19] may be attributed to: (i) The subjectivity of the drop weight test is due to the visual identification of the first crack and failure, which may occur in any direction [16]. (ii) The impact strength of concrete is based on a single point, which may occur either, on a tough particle of coarse aggregate or fibre or matrix or on a soft area of mortar [17] and (iii) Drop hammer being operated manually and it is difficult to control the height of fall of drop hammer exactly; hence the test results would also be greatly influenced by man made errors. Several methods of statistical analysis were performed by different researchers to analyse the variations in drop weight test results [16-19]. The ACI committee 544 drop weight test with its present procedures and recommendations ought not to be considered as a reliable test, which makes the reliability analysis indispensable for evaluating the impact strength for its safe utilization in design of structures subjected to impact loads.

## 2. Research significance

To the authors' best knowledge; there is only one study reporting (Murali et al. 2014) impact failure energy of plain concrete and FRC in terms of reliability, using the graphical method of two parameter Weibull distribution. Hence, in this study the statistical/reliability analysis using two parameter Weibull distribution was carried out extensively on Mastali and Dalvand [19] experimental results using five different methods. So far, the reliability analysis on the impact strength of self-compacting concrete reinforced with recycled CFRP pieces have not been investigated by (Two parameter Weibull distribution) the methods namely LS regression of Y on X, LS regression of X on Y, empirical method, energy factor pattern method and graphical method. Furthermore, few equations were developed to predict and correlate the impact strength of self-compacting concrete made by recycled CFRP pieces using the reliability analysis data. Six reliable statistical indicators namely RMSE, MAPE, MAD, IAE, RRMSE and RPE have been used to validate the developed equations for determining impact strength of self-compacting concrete in terms of reliability function, which has not been performed by earlier researchers. The reliability analysis data would provide a better understanding of the impact strength of reinforced self-compacting concrete while the developed equations are an ideal tool for assessing the impact strength.

## 3. Numerical methods for determining the Weibull parameters

Table 1, shows that impact strength is a random variable and in order to evaluate the accurate impact strength of FRC it is necessary to analyse the results data, statistically [16-20]. The Weibull distribution is broadly used in reliability studies, in recent years several modifications have been developed which greatly enlarged the applications [21, 22]. Based on impact strength test results Weibull distribution can be described as a probability density function F(N) of two-parameter distribution has been shown in equation (1) [20].

$$F(N) = 1 - \exp[-(\tfrac{N}{\alpha})]^{\gamma} \qquad (1)$$

Where γ and α are the shape and scale parameters respectively and N is the impact strength. For estimating the parameters of the Weibull distribution five numerical methods are used. Taking double logarithmic transformation on both sides of the equation (1) can be linearized, i.e.,

$$ln[-ln(1 - F(N))] = \gamma \ln(N) - \gamma \ln(\alpha) \qquad (2)$$

For sample, equation (2) becomes

$$ln[-ln(1 - F_i)] = \gamma \ln N_{(i)} - \gamma \ln(\alpha) \qquad (3)$$

Where i is the order number of failure and $F_i$ denotes the non-parametric estimator of F(N). Several empirical survivorship function formulas for $F_i$ have been proposed in the literature [23]. The commonly used ones are Bernard's median-rank estimator [24].

Bernard's median rank estimator

$$F_i = 1 - \frac{i-0.3}{k+0.4} \qquad (4)$$

In equation (3) setting $Y = ln[-ln(1 - F_i)]$ and $\mathrm{X} = \ln N_{(i)}$, it is transformed into

$$Y = \gamma\mathrm{X} - \gamma \ln(\alpha) \qquad (5)$$

**Table 1**
Experimental impact test results of (Mastali and Dalvand 2016-b).

| S. No | CFRP-0.25 | | CFRP-0.75 | | CFRP-1.25 | |
|---|---|---|---|---|---|---|
| | FC | UC | FC | UC | FC | UC |
| 1 | 23 | 30 | 72 | 110 | 126 | 151 |
| 2 | 56 | 61 | 47 | 66 | 75 | 97 |
| 3 | 30 | 44 | 74 | 82 | 114 | 132 |
| 4 | 50 | 59 | 31 | 39 | 61 | 83 |
| 5 | 22 | 29 | 56 | 75 | 30 | 40 |
| 6 | 61 | 66 | 52 | 62 | 105 | 121 |
| 7 | 42 | 47 | 8 | 104 | 97 | 124 |
| 8 | 65 | 90 | 46 | 58 | 119 | 165 |
| 9 | 81 | 90 | 49 | 57 | 33 | 45 |
| 10 | 42 | 47 | 60 | 69 | 91 | 109 |
| 11 | 44 | 52 | 74 | 106 | 51 | 73 |
| 12 | 35 | 49 | 67 | 78 | 64 | 94 |
| 13 | 36 | 42 | 40 | 48 | 77 | 107 |
| 14 | 24 | 29 | 77 | 109 | 37 | 51 |
| 15 | 56 | 70 | 40 | 54 | 78 | 96 |
| 16 | 28 | 34 | 46 | 68 | 116 | 144 |
| 17 | 32 | 39 | 28 | 40 | 78 | 94 |
| 18 | 25 | 31 | 69 | 85 | 134 | 187 |
| 19 | 49 | 37 | 40 | 52 | 46 | 66 |
| 20 | 46 | 49 | 121 | 159 | 83 | 136 |
| 21 | 51 | 63 | 104 | 133 | 45 | 86 |
| 22 | 61 | 64 | 98 | 119 | 123 | 183 |
| 23 | 9 | 19 | 20 | 25 | 92 | 135 |
| 24 | 40 | 45 | 58 | 64 | 48 | 71 |
| 25 | 63 | 87 | 28 | 33 | 27 | 34 |
| 26 | 32 | 34 | 52 | 61 | 134 | 16 |
| 27 | 79 | 93 | 65 | 71 | 102 | 125 |
| 28 | 39 | 42 | 69 | 87 | 54 | 59 |
| 29 | 55 | 76 | 62 | 83 | 54 | 70 |
| 30 | 38 | 50 | 55 | 64 | 144 | 174 |
| 31 | 37 | 40 | 100 | 126 | 112 | 129 |
| 32 | 30 | 37 | 83 | 114 | 36 | 48 |
| 33 | 47 | 52 | 6 | 86 | 73 | 99 |
| 34 | 48 | 58 | 45 | 62 | 126 | 141 |
| 35 | 42 | 45 | 36 | 49 | 51 | 73 |
| 36 | 23 | 34 | 80 | 96 | 72 | 101 |
| 37 | 41 | 56 | 47 | 66 | 60 | 83 |
| 38 | 46 | 51 | 42 | 59 | 69 | 83 |
| 39 | 28 | 32 | 48 | 54 | 57 | 76 |
| 40 | 47 | 49 | 61 | 87 | 65 | 94 |

FC: First crack impact strength.
UC: Ultimate crack impact strength.

The impact strength $N_R$ in terms of probability of survival (R) i.e, reliability [25].

$$N_R = \alpha(-\ln(R)^{\frac{1}{\gamma}}) \quad (6)$$

### 3.1. Estimators of LS Y on X

The least square method is one of the widely-used method to calculate Weibull parameter, and when it is used while modeling an experiment of a phenomenon, it can give an estimation of the parameters. When using least square method, the sum of the squares of the deviations S is defined [26, 27] as below.

$$S = \sum_{i=1}^{n}[Y_i - (\gamma_0 - \gamma_1 X_i)]^2 \quad (7)$$

Where $\gamma_0$ and $\gamma_1$are the least squares estimators' that minimize the error sum of squares.

Let $a_{yx}$ and $b_{yx}$ denote the estimators of intercept and γ.

$$b_{yx} = \frac{\sum_{i=1}^{n}[(X_i-\bar{X})(Y_i-\bar{Y})]}{\sum_{i=1}^{n}(X_i-X)^2} \quad (8)$$

$$a_{yx} = \bar{Y} - b_{yx}\bar{X} \quad (9)$$

$$\bar{X}_i = \sum_{I=1}^{n} X_i/n \quad (10)$$

$$\bar{Y}_i = \sum_{I=1}^{n} Y_i/n \quad (11)$$

### 3.2 Estimator of LS X on Y

The estimating equation of LS X on Y can be obtained in a similar approach and the formula is given as [26].

$$S = \sum_{i=1}^{n}[X_i - (\frac{1}{\gamma Y_i} + \ln \propto)]^2 \quad (12)$$

$$b_{xy} = \frac{\sum_{i=1}^{n}(Y_i-\bar{Y})^2}{\sum_{i=1}^{n}[(X_i-X)(Y_i-\bar{Y})]} \quad (13)$$

$$a_{yx} = \bar{Y}_i - b_{xy}\bar{X}_i \quad (14)$$

$$\alpha = \exp - (\text{Slope}/\text{ intercept}) \quad (15)$$

The $\bar{X}_i$ and $\bar{Y}_i$ in LS X on Y is calculated using the equations (10 &11). Where, $X_i$=ln(FC), $Y_i$=ln(-ln(1-$F_i$)), $a_{yx}$(Intercept) and $b_{yx}$(γ).

### 3.3. Empirical method

The empirical method is considered as a special case of the moment method and was introduced by [28, 29] where the Weibull parameters γ and α are given by the equations shown below as [28 – 30, 32]. The gamma function is defined by equation (18), and is closely related to the factorial.

$$\gamma = \left(\frac{\sigma}{\bar{N}}\right)^{-1.086} \quad (16)$$

$$\bar{N} = \alpha\Gamma\,(1 + 1/\gamma) \quad (17)$$

$$\Gamma(x) = \int_0^\infty t^{x-1}\exp(-t)\,dt \quad (18)$$

### 3.4 Energy pattern factor method

The energy pattern factor method is related to the averaged data of impact strength and is defined by the following equations [31].

$$Epf = \frac{\overline{N^3}}{\bar{N}^3} \quad (19)$$

$$\gamma = 1 + \frac{3.69}{(Epf)^2} \quad (20)$$

where Epf is the energy pattern factor and the gamma function is defined by the equation (18).

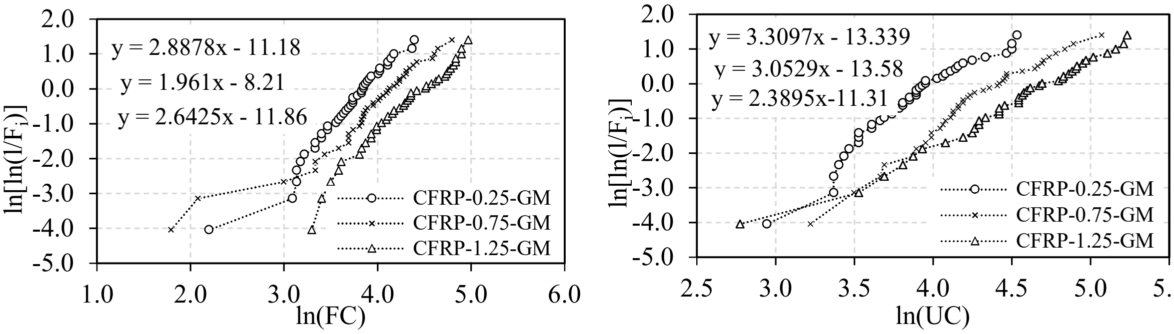


Fig. 1- Weibull lines from graphical method.

**Table 2**

Shape and scale parameter predicted by five methods.

| Method | Parameters | CFRP-0.25 | | CFRP-0.75 | | CFRP-1.25 | |
|---|---|---|---|---|---|---|---|
| | | FC | UC | FC | UC | FC | UC |
| LS-YX | γ | 2.89 | 3.31 | 1.96 | 3.05 | 2.64 | 2.39 |
| | α | 48.01 | 56.28 | 65.85 | 85.48 | 88.86 | 113.66 |
| LS-XY | γ | 3.00 | 3.51 | 2.12 | 3.14 | 2.72 | 2.43 |
| | α | 47.67 | 55.75 | 64.50 | 85.04 | 88.29 | 113.23 |
| EM | γ | 3.01 | 3.03 | 2.45 | 2.85 | 2.58 | 2.59 |
| | α | 47.67 | 56.58 | 63.60 | 85.85 | 88.94 | 112.46 |
| EPM | γ | 2.87 | 2.84 | 2.46 | 2.72 | 2.57 | 2.59 |
| | α | 47.77 | 56.73 | 63.59 | 86.00 | 88.95 | 112.46 |
| GM | γ | 2.88 | 3.31 | 1.96 | 3.05 | 2.64 | 2.39 |
| | α | 48.01 | 56.28 | 65.85 | 85.47 | 88.85 | 113.65 |

### *3.5. Graphical method*

The graphical method is attained using the cumulative distribution function as shown in equation (1). In this method impact strength are interpolated based upon the linear regression using a concept of least square. For this aim, the impact strength data should be sorted into bins first. By taking double logarithmic transformation of equation (1), the equation for the graphical method is obtained as shown in equation (2) [17, 20]. The following processes were carried out to obtain parameters and draw Weibull reliability distribution lines.

1. The impact strength (FC and UC) earlier researcher [19] results were used as input parameter.
2. Serial number was given for each value (i = 1, 2, 3 . . . n).
4. For each value the $F_i$ was calculated by using the equation (4).
5. ln[ln(l/$F_i$)] values were calculated for each value and plotted in Y-axis.
6. ln(FC and UC) values were calculated for each value and plotted in X-axis.
7. Further the regression analysis was carried out using MS excel.
8. The graph of ln[FC and UC] and . ln[ln(l/$F_i$)] values were drawn as shown in Figure [1].
9. The slope and intercept of the line was calculated from Figure [1]. Using the slope of line values of the shape parameters γ and α can be calculated as $\alpha=\exp^{-(\text{Slope}/\ \text{intercept})}$. The above processes were carried out for all the mixtures in order to find the parameters 'α' and 'γ'.

### 4. Results and discussion

The shape parameter and scale parameter calculated using five methods of Weibull distribution are summarized in Table 2. The shape parameter (γ) values for the CFRP-0.25-FC specimens were 2.89, 3.00, 3.01, 2.87 and 2.88 using LS Y on X, LS X on Y, EM, EPM, GM respectively. It can be noted from Table-2 that there is a deviation of shape parameter (γ) value of CFRP-0.25-FC specimens using LS-X on Y and EM method which is about 4.8 % when compared to the LS-Y on X, EPM and GM. Similarly, the scale parameter (α) values for the CFRP-0.25-FC specimens were 48.01, 47.67, 47.67, 47.77 and 48.01 using LS Y on X, LS X on Y, EM, EPM, GM respectively. The deviation observed in the scale parameter (α) value of CFRP-0.25-FC specimens using LS-X on Y and GM method is about 0.7% when compared to LS-X on Y, EPM and EM. A similar trend was observed in CFRP-1.25-FC specimens. Also, it can be observed from Table 2 that, in each mix the shape parameter and scale parameter values calculated using LS Y on X, LS X on Y, EM, EPM, GM were nearly similar and overlapping each other except the CFRP-0.75-FC specimens. Hence, in all the mixes the shape and scale parameter values calculated using LS- X

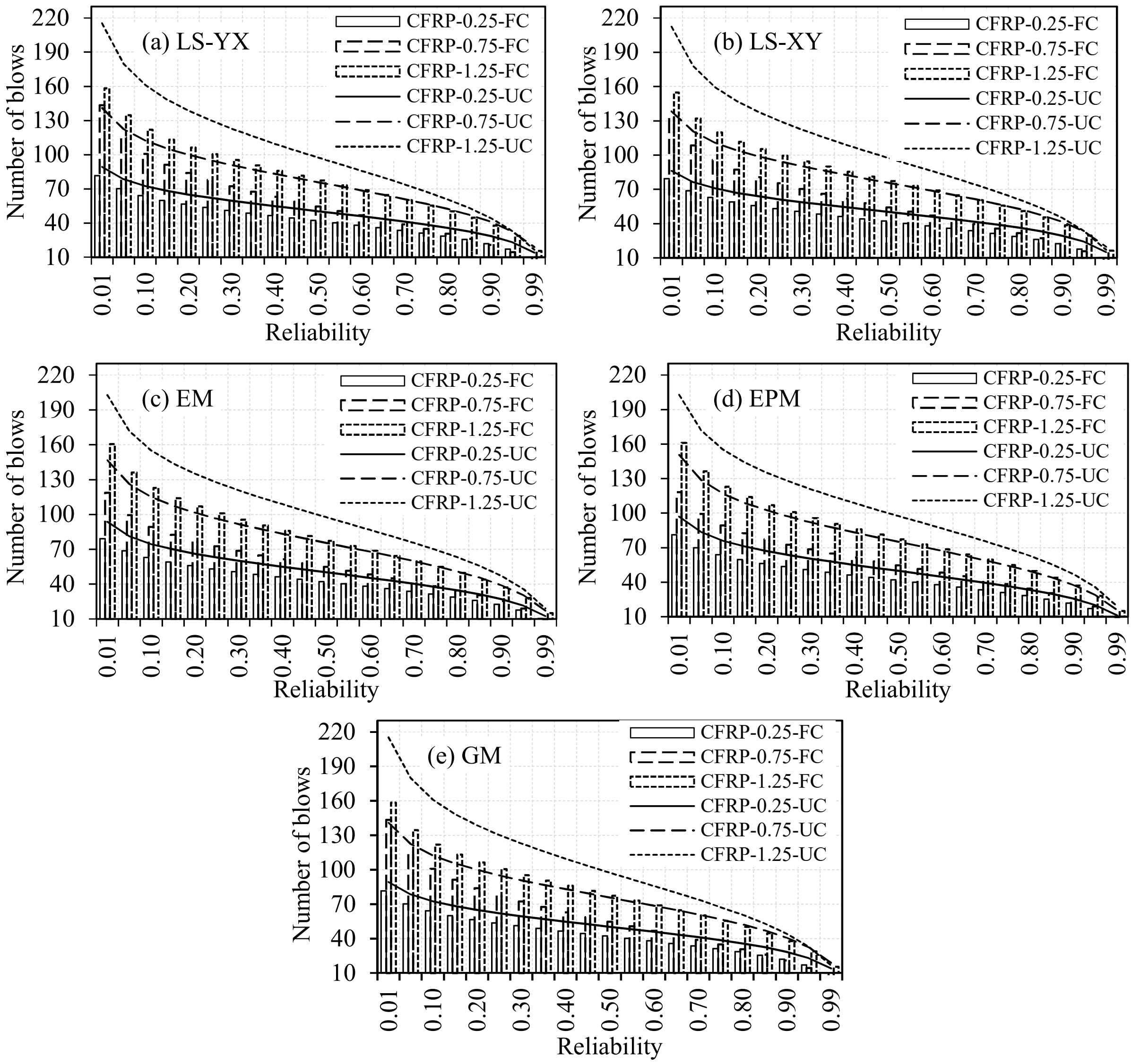

Fig. 2 - Impact strength (FC and UC) interms of reliability of different methods.

on Y, EM and EPM methods are precise [27, 33] and therefore it is fully adequate to estimate the Weibull parameters with good accuracy. These Weibull parameters were used for evaluating the impact strength in terms of reliability.

The impact strength in terms of reliability is calculated by using equation (6) and the plot for each mix using five methods are illustrated in Figures 2 (a)-(e). The reliability curve in Figure 2 (a) shows that impact strength value in terms of reliability for CFRP-0.25 specimens is roughly less than or equal to 81 (Number of blows). 0.90 reliability level is considered for more certain assessment and when this value is substituted in equation (6) along with the Weibull parameters, the obtained impact strength (FC) values in terms of reliability for the CFRP-0.25 specimens were 22, 23, 23, 22 and 22 corresponding to LS-Y on X, LS-X on Y, EM, EPM and GM methods respectively. Similarly, at 0.90 reliability level the impact strength (UC) values in terms of reliability for LS-Y on X, LS-X on Y, EM, EPM and GM were 29, 29, 27, 26 and 29 respectively. On the other side, considering 0.1 reliability level, impact strength values in terms of reliability for LS-Y on X, LS-X on Y, EM, EPM and GM were 64, 63, 63, 64 and 64 respectively in case of FC and were 72, 71, 74, 76 and 72 respectively in case of UC for the mix CFRP-0.25. In the view of large scatter in drop weight impact test results, Weibull distribution method becomes a best choice for describing the impact strength as it discards taking the average of experimental test results. In this respect, Weibull distribution facilitate the designers to describe the impact strength in terms of a reliability level [32].

### *4.1. Impact strength predictions*

The regression analysis was carried out based on impact strength interms of and reliability level is shown in Figure 3. A strong linear relationship was obtained between the FC impact strength and reliability level for CFRP-0.25, CFRP-0.75 and CFRP-1.25 specimens, with the coefficients of determination ($R^2$) values hitting 0.964, 0.934, and 0.960 respectively. Simillarly for the UC impact

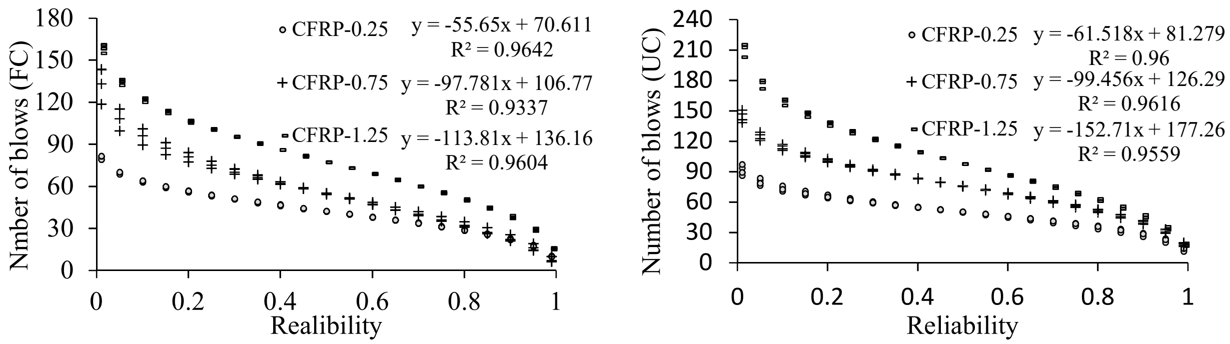


Fig. 3 - Scatter diagram of impact strength with fitted regression line.

strength, the $R^2$ values hitting 0.960, 0.962, and 0.956 for CFRP-0.25, CFRP-0.75 and CFRP-1.25 specimens respectively.

The positive linear relationship between the FC and UC were described using the linear regression model as follows.

$[CFRP-0.25-FC]=-55.65\ X+70.61$ (21)

$[CFRP-0.75-FC]=-97.78\ X+106.77$ (22)

$[CFRP-1.25-FC]=-113.81\ X+136.16$ (23)

$[CFRP-0.25-UC]=-61.52\ X+81.28$ (24)

$[CFRP-0.75-UC]=-99.46\ X+126.29$ (25)

$[CFRP-1.25-UC]=-152.71\ X+177.26$ (26)

Therefore, the equations derived from linear regression analysis may efficiently be used to represent the relationship between the FC and UC with reliability at 95% prediction interval along with the fitted regression model graphically, as shown in Figure [3]. The obtained correlation coefficient (R) from the equations (21 to 26) for CFRP-0.25, CFRP-0.75 and CFRP-1.25 specimens were 0.982, 0.966 and 0.980 in case of FC and 0.979, 0.998 and 0.977 in case of UC. The developed equations that possessed the $R^2$ value higher than 0.7 was considered as equitable by most statisticians [33]. According to [34], the validity of appropriate equations was also based on the values of correlation coefficient (R) and determination coefficient ($R^2$). However, the value $R^2$ alone cannot validate the developed equations [35] and these results have to be combined with several reliable statistical indicators to evaluate their accuracy.

### *4.2. Statistical indicators used for performance evaluation*

To assess the performance of developed equations for estimation of impact strength (FC and UC) in terms of reliability, different statistical approaches including six reliable statistical indicators have been used in this study. Six statistical parameters consisting root mean squared error, mean absolute percentage error, mean absolute deviation, integral absolute error, relative root mean square error and relative percentage error have been employed to offer an appropriate comparative assessment. In the following, a brief description of the considered statistical indicators is offered.

#### *4.2.1. Root Mean Squared Error (RMSE)*

RMSE identifies the model's accuracy by comparing the variation between experimental value ($E_V$) and predicted values ($P_V$) and it can be determined from equation (27) [36]. If RMSE has positive value and it is nearer to zero it indicates, a good fit [36].

$$\mathrm{RMSE}=\sqrt{\frac{1}{n}\sum_{i=1}^{n}(E_V-P_V)^2} \quad (27)$$

#### *4.2.2. Mean absolute percentage error (MAPE)*

The MAPE shows the mean absolute percentage difference between $E_V$ and $P_V$. The MAPE is calculated by [35]:

$$\mathrm{MAPE}=\left(\frac{1}{n}\sum_{i=1}^{n}\left|\frac{E_V-P_V}{E_V}\right|\right)\times 100\% \quad (28)$$

#### *4.2.3. Mean Absolute Deviation (MAD)*

MAD denotes the average quantity of errors between the $E_V$ and $P_V$. The MAD is calculated as follows [14].

$$\mathrm{MAD}=\frac{\sum|E_V-P_V|}{n} \quad (29)$$

#### *4.2.4. Integral Absolute Error (IAE)*

IAE is employed to evaluate the deviation between the $E_V$ and $P_V$ curves. This is written as [19].

$$\mathrm{IAE}=\sum\frac{[(E_V-P_V)^2]^{1/2}}{E_V}\times 100 \quad (30)$$

#### *4.2.5. Relative Root Mean Square Error (RRMSE)*

The RRMSE is obtained by dividing the RMSE to the average of impact strength obtained by $E_V$ as follows [31]:

$$\mathrm{RRMSE}=\frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(E_V-P_V)}}{\frac{1}{n}\sum_{i=1}^{n}E_V} \quad (31)$$

Different ranges of RRMSE can be well-defined to signify the models' precision as [37]: Excellent for RRMSE < 10%; Good for 10% < RRMSE < 20%; Fair for 20% < RRMSE < 30%; Poor for RRMSE > 30%.

**Table 3**

Comparison of results obtained from six statistical indicators.

| Mix Id | RMSE | MAD | MAPE | IAE | RRMSE | RPE | $R^2$ |
|---|---|---|---|---|---|---|---|
| CFRP-0.25-FC | 0.2404 | -0.0380 | -0.0008 | 0.0003 | 0.2141 | -0.0338 | 0.9642 |
| CFRP-0.75-FC | 0.0448 | -0.0071 | -0.0001 | 0.0000 | 0.0295 | -0.0047 | 0.9337 |
| CFRP-1.25-FC | 0.0442 | -0.0070 | -0.0001 | 0.0000 | 0.0212 | -0.0034 | 0.9604 |
| CFRP-0.25-UC | 0.0070 | 0.0011 | 0.0000 | 0.0000 | 0.0053 | 0.0008 | 0.9600 |
| CFRP-0.75-UC | 0.0231 | -0.0036 | 0.0000 | 0.0000 | 0.0115 | -0.0018 | 0.9616 |
| CFRP-1.25-UC | 0.0236 | -0.0037 | 0.0000 | 0.0000 | 0.0089 | -0.0014 | 0.9559 |

### *4.2.6. Relative Percentage Error (RPE)*

The RPE shows the percentage deviation between the $E_V$ and $P_V$ and its values ranging between +10% and -10% are usually considered acceptable [38]. RPE is defined as:

$$\mathrm{RPE} = \frac{(E_V - P_V)}{E_V} \text{ x } 100 \quad (32)$$

The most accurate of developed equation will be the one which has the highest value for the $R^2$ and the lowest values for RMSE, MAD, MAPE, IAE, RRMSE and RPE. The predicted value from the developed equation was compared to the actual data. Based on the statistical analysis the developed equations have the highest $R^2$ and the lowest values for RMSE, MAD, MAPE, IAE, RRMSE and RPE for all the mixes is shown in Table-3, which ensures that it is the most accurate validation for the developed equations to predict the impact strength in terms of reliability.

## 5. Conclusions

It is critical to choose the design values based on the experimental results of drop weight tests due to its lack of accuracy and it might maximise the probability of failure. In this paper, Weibull distribution was employed to analyse the drop weight test results and this method is an efficient and practical method which can be used for composite materials too. By using Weibull parameters, the impact strength of self-compacting concrete reinforced with recycled CFRP pieces was presented in terms of reliability function/Probability of survival. Analysis has yielded the following conclusions:

- Reliability analysis using Weibull distribution was performed with the help of five different methods. Analysis suggested that the LS- X on Y, EM and EPM are more effective to estimate the Weibull parameters accurately than LS- Y on X and GM.
- Considering the 0.90 reliability level, the impact strength (FC) values for the mix CFRP-0.25 were 22, 23, 23, 22 and 22 in case of LS-Y on X, LS-X on Y, EM, EPM and GM respectively. By introducing Weibull distribution for analysing the experimental test results, it led to elimination of taking the average. In this respect, Weibull distribution facilitate the designers to describe the impact strength in terms of a reliability level.
- The developed linear regression equations were precise to evaluate impact strength of self-compacting concrete reinforced with recycled CFRP pieces at FC and UC, the coefficients of determination ($R^2$) values hitting 0.964, 0.934, and 0.960 in case of FC and 0.960, 0.962, and 0.956 in case of UC for CFRP-0.25, CFRP-0.75 and CFRP-1.25 specimens respectively.
- The validity of the developed linear regression equation for RMSE, MAD, MAPE, IAE, RRMSE and RPE for all the mixes had the values closer to zero and highest values of $R^2$ nearer to 1 which indicates a good fit and higher accuracy. Lastly, the Weibull distribution was employed here to model an impact strength property and reliability analysis, but it can also be used in areas with similar uncertainties.

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