

# ANALIZE STATISTICE ȘI MODELE PREDICTIVE PENTRU CERAMICI COMPOZITE DENSE DE TIP Y-ZrO<sub>2</sub> – Al<sub>2</sub>O<sub>3</sub> STATISTICAL ANALYSIS AND PREDICTION MODELS FOR YTTRIA-STABILIZED ZrO<sub>2</sub>-Al<sub>2</sub>O<sub>3</sub> DENSE COMPOSITE CERAMICS

ȘTEFANIA STOLERIU, ALEXANDRU CONSTANTINESCU\*, ADRIAN VOLCEANOV

Universitatea POLITEHNICA București, Str. G. Polizu nr. 1, 011061, sect. 1, București, România

*The most common objective of the statistical analysis is to explain the influence of random variations and events. Perhaps the most important objective is to manage simultaneous and combined effects of several influencing factors, when those factors can not be easily controlled in the experimental frame.*

*Multivariate Analysis – MVA is based on the principle of multidimensional statistics, which involves observation and analysis of several statistical variables at a time. This analysis is used to study several dimensions simultaneously, taking into account the effects of all variables on the characteristics of interest [1-4].*

*By introducing ceramic properties of some dense composite ceramics, zirconia – alumina type, with oxide additives for densification as independent variables, it was obtained a quality model. The model proved that it is necessary a more rigorous control of these properties in order to improve the compressive strength [5-6].*

*The developed statistical models are consistent with existing assertions in this area - they are viable, but also offers new insights - of interest from the point of view of optimizing the composition and properties of specific ceramics.*

*Cel mai cunoscut obiectiv al analizei statistice este acela de a explica influența variațiilor aleatorii. Probabil cel mai important obiectiv este acela de a descurca efectele simultane și combinate ale mai multor factori de influență, în situații în care acei factori nu pot fi ușor controlați în cadrul experimentului.*

*Analiza multidimensională (MVA - Multivariate Analysis) se bazează pe principiul statisticii multidimensionale, care implică observarea și analizarea mai multor variabile statistice într-un moment. Această analiză este folosită pentru a studia mai multe dimensiuni concomitent, ținând cont de efectele tuturor variabilelor asupra caracteristicilor de interes.*

*Prin introducerea proprietăților ceramice ale unor ceramici compozite dense de tip zirconă – alumină, cu aditivi oxidici pentru densificare, ca variabile independente obținem un model calitativ superior. Acest model dovedește că este necesar un control mai riguros al acestor proprietăți pentru a îmbunătăți rezistența mecanică la compresiune.*

*Modelele statistice create sunt în concordanță cu aserțiunile deja existente în acest domeniu - sunt viabile, dar oferă și perspective noi - prezintă interes, din punctul de vedere al optimizării atât a compozițiilor cât și a proprietăților specifice ceramicilor.*

**Keywords:** statistics, ZTA properties, prediction, dopants

## 1. Multivariate analysis

Statistical analysis is seen by experts as a necessary evil to public scientific articles, as another way to influence friends and enemies to a certain perspective, but there is a group saying that it is an irrelevant deviation from the real science. Statistical tools serve these purposes only to few people. However, in the right hands, they can be very useful to scientific purposes, which is an important step toward understanding the mechanisms underlying those scientific data. Another positive aspect of the statistical analysis is to allow optimization of processes and compositions specific for certain materials or systems.

The most common objective of the statistical analysis is to explain the influence of random variations.

Perhaps the most important objective is that to manage simultaneous and combined effects of several influence factors in the situations in which those factors can not be easily controlled within the experiment [1 - 7].

Multivariate Analysis – MVA is based on the principle of multidimensional statistics, which involves observation and analysis of several statistical variables at a time. This analysis is used to study several dimensions simultaneously, taking into account the effects of all variables on the characteristics of interest [8].

### 1.1. Prediction

In the scientific context, a prediction is a rigorous estimate, usually quantitative, anticipating what will happen under certain conditions. This is done through repeatable experiments or observational studies.

\* Autor corespondent/Corresponding author,  
Tel.: + 40 21 402.39.97, e-mail: [constantinescu\\_alexandru\\_andrei@yahoo.com](mailto:constantinescu_alexandru_andrei@yahoo.com)

A scientific theory whose assertions are not consistent with existing observations and evidence will be denied. Predictions remain untested theories containing proto-science until their test methods become known to community. In addition, when new theories generate many predictions, they are very popular because they can be confirmed or refuted quickly and easily. In most scientific fields, desirable theories are those that predict a large number of events from few basic principles.

### 1.2. Methods

In statistics, correlation (often measured as a correlation coefficient) indicates the direction and strength of linear relationship between two random variables. The capability of multidimensional analysis procedure determines the probability that elements characterized by two or more variables falls in special technical conditions certified. When variables are correlated is important to consider their overall behavior, because treating each variable separately one can obtain deceptive information on overall system capability.

Method of least squares builds a statistical model linking X independent variable with Y dependent variable. The procedure is very useful when there are several factors and the main objective is the prediction of the dependent variables (response variables). Multiple regression procedure determines a statistical model that describes the impact of two or more quantitative factors X on a dependent variable Y. The model can be used to make predictions, but also to determine the limits of certainty. Residual elements (outliers) can be identified and also discovered the values / data sets with a major influence [8-10].

Canonical correlations help to identify associations between two sets of variables by finding linear combinations of the two sets of variables showing strong correlations. Pair of linear combinations with the strongest correlations forms the first set of canonical variables. The second set is the pair of linear combinations showing the strongest canonical correlation between combinations that were not correlated in the first set. Usually we can express quantitatively the relationship between the two sets.

The multivariable analysis has the following steps [11]:

- a. *Initial data* – data processing and sorting in a suitable database.
- b. *Data cleaning* – the removal of dataset samples that are inadequate or contains missing values.
- c. *Visual analysis* – allows preliminary correlations and outlier detection.
- d. *Correlations* - main factors in selecting variables by using methods such as MVA, PLS, multiple regressions.
- e. *Selection of variables* – choosing a larger number of variables for a complete process, but also to have a correct statistical prediction and removing similar variables. This is done using correlation methods - in our case multiple regression.
- f. *Group analysis* – helps to remove exceptional values (outliers), thus improving the prediction, and confirm if the prediction is indeed viable and valid.

Table 1

Variables used for analysis models and statistical prediction  
Variabilele folosite pentru elaborarea modelelor matematice și analiză statistică

Code / Cod	Signification / Definiție	Minimum / maximum values Valorile minime / maxime
<b>Independent variables / Variabile independente</b>		
Al <sub>2</sub> O <sub>3</sub> (%)	Mass proportion of Al <sub>2</sub> O <sub>3</sub> / <i>Proportia masică de Al<sub>2</sub>O<sub>3</sub></i>	20/80
ZrO <sub>2</sub> * (%)	Mass proportion of ZrO <sub>2</sub> stabilized with 8% Y <sub>2</sub> O <sub>3</sub> / <i>Proportia masică de ZrO<sub>2</sub> stabilizat cu 8% Y<sub>2</sub>O<sub>3</sub></i>	20/80
Temp. (°C)	Heat treatment temperature / <i>Temperatura de tratament termic</i>	1400/1600
Soaking time (min.)	Soaking time / <i>Palierul de menținere la temperatura maximă</i>	180/360
ZnO (%)	Mass proportion of ZnO / <i>Proportia masică de ZnO</i>	0/2
CuO (%)	Mass proportion of CuO / <i>Proportia masică de CuO</i>	0/2
MnO <sub>2</sub> (%)	Mass proportion of MnO <sub>2</sub> / <i>Proportia masică de MnO<sub>2</sub></i>	0/2
TiO <sub>2</sub> (%)	Mass proportion of TiO <sub>2</sub> / <i>Proportia masică de TiO<sub>2</sub></i>	0/2
<b>Dependent variables / Variabile dependente</b>		
Shrinkage (%)	Shrinkage / <i>Contrație</i>	40.62/63.96
ρ <sub>a</sub> (g/cm <sup>3</sup> )	Apparent density / <i>Densitatea aparentă</i>	2.8/5.38
ρ <sub>r</sub> (%)	Relative density / <i>Densitatea relativă</i>	67.72/99.71
Abs. (%)	Absorption / <i>Absorbția</i>	0.09/15.51
Pd (%)	Open porosity / <i>Porozitatea deschisă</i>	0.97/50.77
Rc (MPa)	Compressive strength / <i>Rezistența mecanică la compresiune</i>	69.41/152.35
E (GPa)	Elasticity, Young's modulus / <i>Modulul de elasticitate Young</i>	0.13/1.95

## 2. Results and discussion

All the experimental data, presented in paper *Yttria-stabilized ZrO<sub>2</sub>-Al<sub>2</sub>O<sub>3</sub> dense composite ceramics. Obtaining and characterization* (published in same journal number) [12] were considered for present multivariable analysis. After initial data processing and sorting, from above mentioned results there have been removed 3 sets of data. Thus, there were developed seven mathematical models that can be further used for prediction (after validation [13,14]) with 15 variables (8 independent variables and 7 dependent variables) and 77 experimental data sets. The considered variables are presented in Table 1.

Logically, it has to remove the variable ZrO<sub>2</sub>

content because it is reversed proportional with variable Al<sub>2</sub>O<sub>3</sub> content, but by doing so it was noticed a decrease of confidence intervals and R<sup>2</sup> values of statistical models of ceramic and mechanical properties between 3 and 9%.

To create these statistical models it has been used the Statgraphics Centurion XVI program [15].

Due to relatively low number of data sets and independent variables (proportion of Al<sub>2</sub>O<sub>3</sub> and ZrO<sub>2</sub>, sintering treatment conditions - temperature and soaking time; oxide additives) have limited opportunities to improve these statistical models.

A good criterion for choosing a model configuration is to have simultaneously R<sup>2</sup> biggest coefficient and smallest prediction errors.

### 2.1. Mathematical models

#### Shrinkage model

Dependent variable: Shrinkage (%)

Independent variables: Al<sub>2</sub>O<sub>3</sub> (%) ZrO<sub>2</sub>\* (%) Temp. (°C), Soaking time (min.) ZnO (%), CuO (%), MnO<sub>2</sub> (%) TiO<sub>2</sub> (%)

Table 2

Mathematical factors for shrinkage model / Factorii matematice pentru modelul matematic al contractiei

Parameter / Parametru	Estimation / Estimare	Standard Error / Eroarea Standard	P value / Valoarea P
CONSTANT	-5.70223	5.09581	0.2671
Al <sub>2</sub> O <sub>3</sub> (%)	0.103075	0.0262134	0.0002
ZrO <sub>2</sub> * (%)	0.011895	0.0278055	0.6709
Temp. (°C)	0.0359412	0.00297234	0.0000
Soaking time (min.) <i>Palierul de tratament termic (min.)</i>	0.0124736	0.00259626	0.0000
ZnO (%)	2.21619	0.343207	0.0000
CuO (%)	-3.23974	0.343207	0.0000
MnO <sub>2</sub> (%)	-0.822973	0.348988	0.0212
TiO <sub>2</sub> (%)	-3.35831	0.348988	0.0000

R<sup>2</sup> = 91.0223%

Estimated standard error = 1.79564

Average error modulus = 1.33611

Shrinkage model equation is:

Shrinkage (%) = -5.70223 + 0.103075·Al<sub>2</sub>O<sub>3</sub> (%) + 0.011865·ZrO<sub>2</sub>\* (%) + 0.0359412·Temp. (°C) + 0.0124736·Soaking time (min.) + 2.21619·ZnO (%) -3.23974·CuO (%) -0.822973·MnO<sub>2</sub> (%) -3.35831·TiO<sub>2</sub> (%)

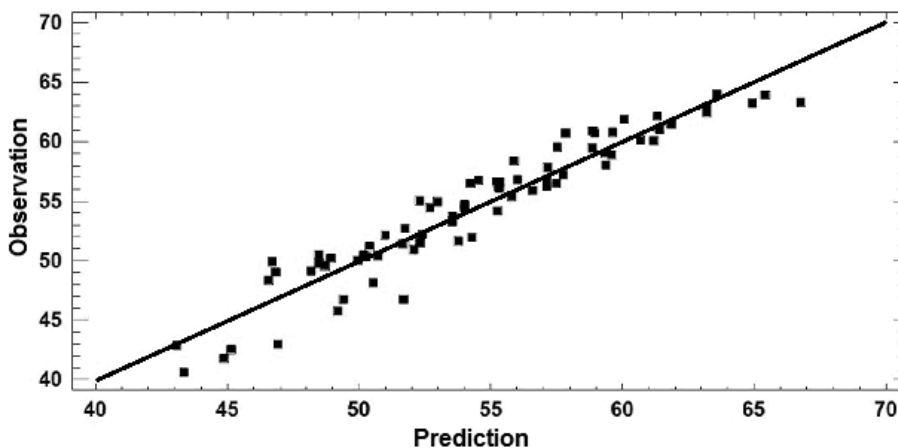


Fig. 1 - Shrinkage model / Modelul matematic pentru contractie.

**Apparent density model**

Dependent variable:  $\rho_a$  (g/cm<sup>3</sup>)

Independent variables: Al<sub>2</sub>O<sub>3</sub> (%) ZrO<sub>2</sub>\* (%) Temp. (°C), Soaking time (min.) ZnO (%), CuO (%), MnO<sub>2</sub> (%) TiO<sub>2</sub> (%)

**Table 3**

Mathematical factors for apparent density model / *Factorii matematice pentru modelul matematic al densității aparente*

Parameter / Parametru	Estimation / Estimare	Standard Error / Eroarea Standard	P value / Valoarea P
CONSTANT	2.82402	0.402741	0.0000
Al <sub>2</sub> O <sub>3</sub> (%)	-0.0306131	0.00207174	0.0000
ZrO <sub>2</sub> * (%)	-0.00257457	0.00219757	0.2455
Temp. (°C)	0.00181384	0.000234915	0.0000
Soaking time (min.) <i>Palierul de tratament termic (min.)</i>	0.000893947	0.000205192	0.0000
ZnO (%)	0.232388	0.0271249	0.0000
CuO (%)	0.0839509	0.0271249	0.0029
MnO <sub>2</sub> (%)	0.170099	0.0275818	0.0000
TiO <sub>2</sub> (%)	-0.0172341	0.0275818	0.5342

R<sup>2</sup> = 96.0923%

Estimated standard error = 0.141916

Average error modulus = 0.110841

Apparent density model equation is:

$$\rho_a \text{ (g/cm}^3\text{)} = 2.82402 - 0.0306131 \cdot \text{Al}_2\text{O}_3 \text{ (%) - 0.00257457} \cdot \text{ZrO}_2^* \text{ (%) + 0.00181384} \cdot \text{Temp. (}^\circ\text{C)} + 0.000893947 \cdot \text{Soaking time (min.) + 0.232388} \cdot \text{ZnO (%) + 0.0839509} \cdot \text{CuO (%) + 0.170099} \cdot \text{MnO}_2 \text{ (%) - 0.0172341} \cdot \text{TiO}_2 \text{ (%)}$$

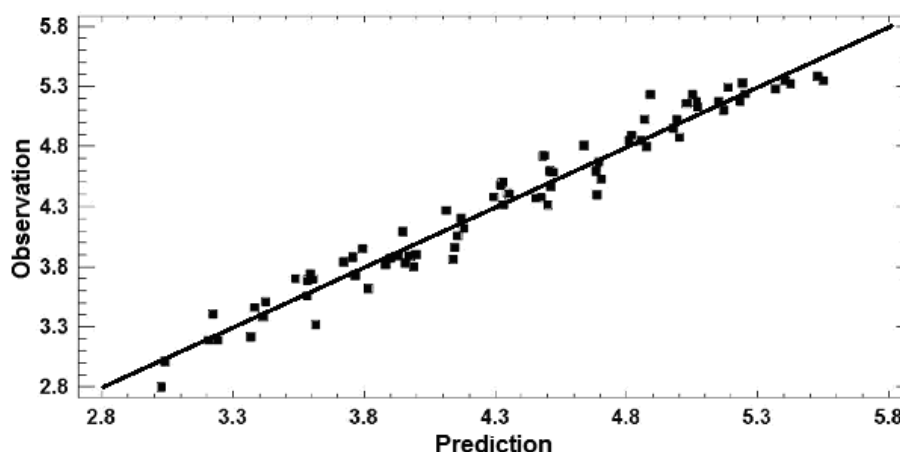


Fig. 2 - The apparent density model / *Modelul matematic pentru densitatea aparentă.*

**Relative density model**

Dependent variable:  $\rho_r$  (%)

Independent variables: Al<sub>2</sub>O<sub>3</sub> (%) ZrO<sub>2</sub>\* (%) Temp. (°C), Soaking time (min.) ZnO (%), CuO (%), MnO<sub>2</sub> (%) TiO<sub>2</sub> (%)

**Table 4**

Mathematical factors for relative density model / *Factorii matematice pentru modelul matematic al densității relative*

Parameter / Parametru	Estimation / Estimare	Standard Error / Eroarea Standard	P value / Valoarea P
CONSTANT	39.6808	8.40648	0.0000
Al <sub>2</sub> O <sub>3</sub> (%)	-0.252222	0.0432439	0.0000
ZrO <sub>2</sub> * (%)	-0.0800385	0.0458703	0.0855
Temp. (°C)	0.0382213	0.00490342	0.0000
Soaking time (min.) <i>Palierul de tratament termic (min.)</i>	0.0201588	0.00428301	0.0000
ZnO (%)	5.38649	0.566183	0.0000
CuO (%)	1.90587	0.566183	0.0013
MnO <sub>2</sub> (%)	4.17262	0.575719	0.0000
TiO <sub>2</sub> (%)	-0.253052	0.575719	0.6617

R<sup>2</sup> = 85.4859%

Estimated standard error = 2.96224

Average error modulus = 2.3375

The relative density model equation is

$$\rho_r (\%) = 39,6808 - 0,252222 \cdot \text{Al}_2\text{O}_3 (\%) - 0,0800385 \cdot \text{ZrO}_2^* (\%) + 0,0382213 \cdot \text{Temp. } (^\circ\text{C}) + 0,0201588 \cdot \text{Soaking time (min.)} + 5,38649 \cdot \text{ZnO } (\%) + 1,90587 \cdot \text{CuO } (\%) + 4,17262 \cdot \text{MnO}_2 (\%) - 0,253052 \cdot \text{TiO}_2 (\%)$$

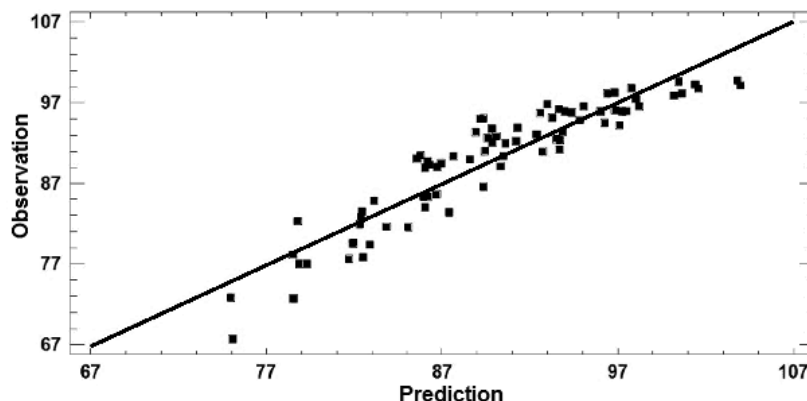


Fig. 3 - The relative density model / Modelul matematic pentru densitatea relativă.

**Absorption model**

Dependent variable: Abs. (%)

Independent variables: Al<sub>2</sub>O<sub>3</sub> (%) ZrO<sub>2</sub>\* (%) Temp. (°C), Soaking time (min.) ZnO (%), CuO (%), MnO<sub>2</sub> (%) TiO<sub>2</sub> (%)

**Table 5**

Mathematical factors for absorption model / Factorii matematici pentru modelul matematic al absorbției

Parameter / Parametru	Estimation / Estimare	Standard Error / Eroarea Standard	P value / Valoarea P
CONSTANT	30.8884	5.71924	0.0000
Al <sub>2</sub> O <sub>3</sub> (%)	0.0940236	0.024204	0.0021
ZrO <sub>2</sub> * (%)	0.0273965	0.0312073	0.3831
Temp. (°C)	-0.0208035	0.00333598	0.0000
Soaking time (min.) Palierul de tratament termic (min.)	-0.00717832	0.00291389	0.0163
ZnO (%)	-1.50282	0.385196	0.0002
CuO (%)	-0.311885	0.385196	0.4209
MnO <sub>2</sub> (%)	-1.18837	0.391683	0.0034
TiO <sub>2</sub> (%)	0.781959	0.391683	0.0499

R<sup>2</sup> = 68,2132%

Estimated standard error = 2.01532

Average error modulus = 1.4882

Absorption model equation is:

$$\text{Abs. } (\%) = 30.8884 + 0.0940236 \cdot \text{Al}_2\text{O}_3 (\%) + 0.0273965 \cdot \text{ZrO}_2^* (\%) - 0.0208035 \cdot \text{Temp. } (^\circ\text{C}) - 0.00717832 \cdot \text{Soaking time (min.)} - 1.50282 \cdot \text{ZnO } (\%) - 0.311885 \cdot \text{CuO } (\%) - 1.18837 \cdot \text{MnO}_2 (\%) + 0.781959 \cdot \text{TiO}_2 (\%)$$

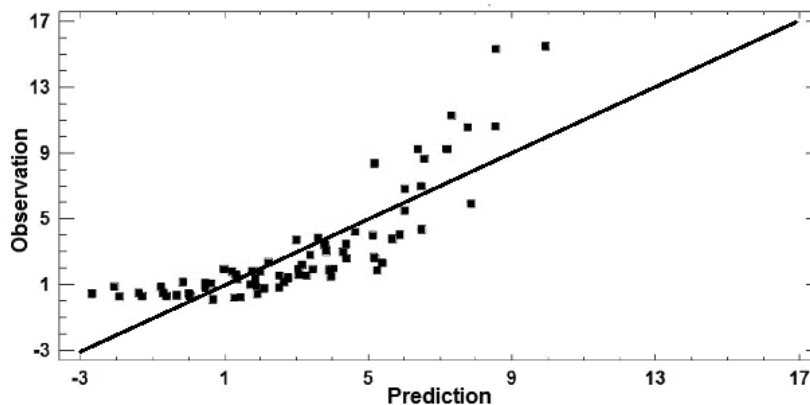


Fig. 4. Absorption model / Modelul matematic pentru absorbție.

**Open porosity model**

Dependent variable: Pd (%)

Independent variables: Al<sub>2</sub>O<sub>3</sub> (%) ZrO<sub>2</sub>\* (%) Temp. (°C), Soaking time (min.) ZnO (%), CuO (%), MnO<sub>2</sub> (%) TiO<sub>2</sub> (%)

**Table 6**

Mathematical factors for open porosity model / *Factorii matematici pentru modelul matematic al porozității deschise*

Parameter / Parametru	Estimation / Estimare	Standard Error / Eroarea Standard	P value / Valoarea P
CONSTANT	118.241	20.2479	0.0000
Al <sub>2</sub> O <sub>3</sub> (%)	0.261719	0.104157	0.0144
ZrO <sub>2</sub> * (%)	0.0617015	0.110483	0.5784
Temp. (°C)	-0.0763003	0.0118104	0.0000
Soaking time (min.) <i>Palierul de tratament termic (min.)</i>	-0.0277221	0.0103161	0.0090
ZnO (%)	-4.351	1.36371	0.0021
CuO (%)	0.794937	1.36371	0.5619
MnO <sub>2</sub> (%)	-3.38545	1.38668	0.0172
TiO <sub>2</sub> (%)	3.56522	1.38668	0.0123

R<sup>2</sup> = 66,6501 %

Estimated standard error = 7.13485

Average error modulus = 4.93547

Open porosity model equation is:

$$Pd (\%) = 118.241 + 0.261719 \cdot Al_2O_3 (\%) + 0.0617015 \cdot ZrO_2^* (\%) - 0.0763003 \cdot Temp. (^\circ C) - 0.0277221 \cdot Soaking\ time (min.) - 4.35 \cdot ZnO (\%) + 0.794937 \cdot CuO (\%) - 3.38545 \cdot MnO_2 (\%) + 3.56522 \cdot TiO_2 (\%)$$

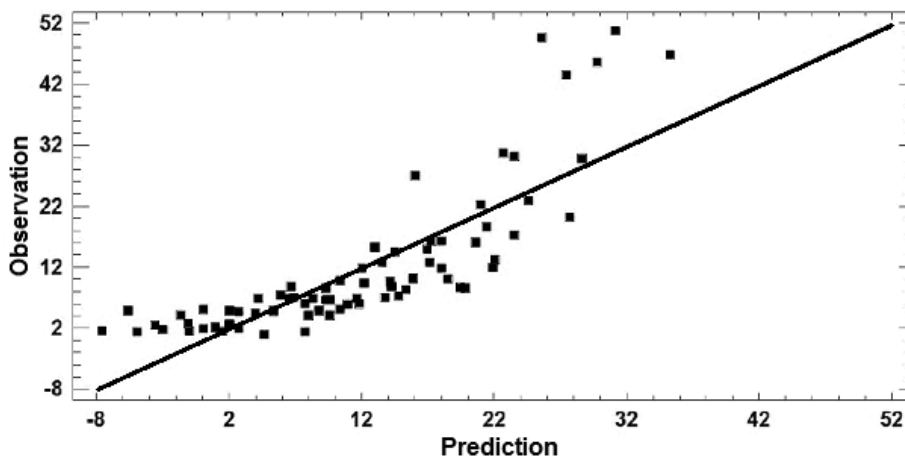


Fig. 5 - Open porosity model / *Modelul matematic pentru porozitatea deschisă.*

**Compressive strength model**

Dependent variable: Rc (MPa)

Independent variables: Al<sub>2</sub>O<sub>3</sub> (%) ZrO<sub>2</sub>\* (%) Temp. (°C), Soaking time (min.) ZnO (%), CuO (%), MnO<sub>2</sub> (%) TiO<sub>2</sub> (%)

**Table 7**

Mathematical factors for compressive strength model  
*Factorii matematici pentru modelul matematic al rezistenței mecanice la compresiune*

Parameter / Parametru	Estimation / Estimare	Standard Error / Eroarea Standard	P value / Valoarea P
CONSTANT	-32.9786	41.8797	0.4337
Al <sub>2</sub> O <sub>3</sub> (%)	-0.525366	0.215434	0.0174
ZrO <sub>2</sub> * (%)	-0.35714	0.228518	0.1227
Temp. (°C)	0.122417	0.024428	0.0000
Soaking time (min.) <i>Palierul de tratament termic (min.)</i>	-0.012758	0.0213372	0.5519
ZnO (%)	7.33005	2.82063	0.0115
CuO (%)	-4.43557	2.82063	0.1205
MnO <sub>2</sub> (%)	4.45182		0.1253
TiO <sub>2</sub> (%)	-8.01118	2.86814	0.0068

R<sup>2</sup> = 55,0366 %

Estimated standard error = 14.7574

Average error modulus = 11.8973

Compressive strength model equation is:

$$R_c \text{ (MPa)} = -32.9786 - 0.525366 \cdot \text{Al}_2\text{O}_3 \text{ (\%)} - 0.35714 \cdot \text{ZrO}_2^* \text{ (\%)} + 0.122417 \cdot \text{Temp. (}^\circ\text{C)} - 0.012758 \cdot \text{Soaking time (min.)} + 7.33005 \cdot \text{ZnO (\%)} - 4.43557 \cdot \text{CuO (\%)} + 4.45182 \cdot \text{MnO}_2 \text{ (\%)} - 8.01118 \cdot \text{TiO}_2 \text{ (\%)}$$

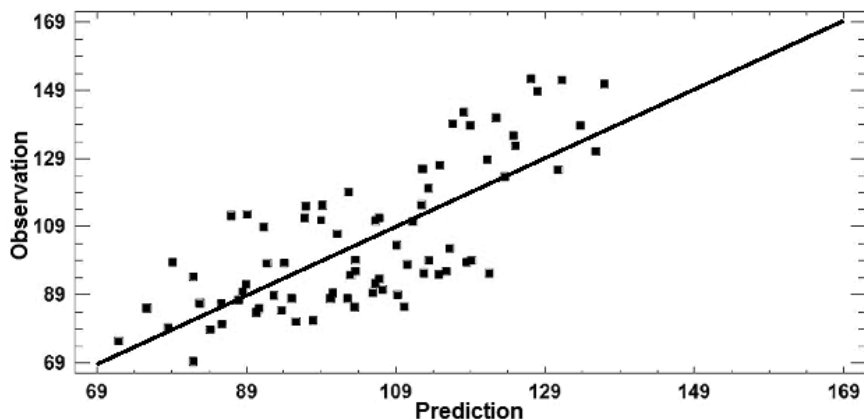


Fig. 6 - Model for compressive strength / Modelul matematic pentru rezistența mecanică la compresiune.

Improved compressive strength model

Dependent variable: Rc (MPa)

Independent variables: Al<sub>2</sub>O<sub>3</sub> (%), ZrO<sub>2</sub>\* (%), Temp. (°C), Soaking time (min.), ZnO (%), CuO (%), MnO<sub>2</sub> (%), TiO<sub>2</sub> (%), Cont. (%), ρ<sub>a</sub> (g/cm<sup>3</sup>), ρ<sub>r</sub> (%), Abs. (%), Pd (%)

Table 8

Mathematical factors for improved compressive strength model  
Factorii matematici pentru modelul matematic îmbunătățit al rezistenței mecanice la compresiune

Parameter / Parametru	Estimation / Estimare	Standard Error / Eroarea Standard	P value / Valoarea P
CONSTANT	-67.584	83.5778	0.4218
Al <sub>2</sub> O <sub>3</sub> (%)	0.143129	0.578371	0.8053
ZrO <sub>2</sub> * (%)	-0.547321	0.217475	0.0144
Temp. (°C)	0.127883	0.0454646	0.0065
Soaking time (min.) Palierul de tratament termic (min.)	-0.00105289	0.0249974	0.9665
ZnO (%)	12.2481	4.40662	0.0072
CuO (%)	2.40802	4.60937	0.6032
MnO <sub>2</sub> (%)	12.0866	3.87691	0.0027
TiO <sub>2</sub> (%)	-4.14578	4.41551	0.3514
Shrinkage (%) Contractție (%)	1.25915	0.956357	0.1927
ρ <sub>a</sub> (g/cm <sup>3</sup> )	59.9319	24.9881	0.0194
ρ <sub>r</sub> (%)	-3.76637	1.32296	0.0060
Abs. (%)	1.47288	2.25492	0.5160
Pd (%)	-0.198794	0.586612	0.7358

R<sup>2</sup> = 64,6937 %

Estimated standard error = 13.5859

Average error modulus = 10.3082

Equation improved compressive strength model is

$$R_c \text{ (MPa)} = 67584 + 0.143129 \cdot \text{Al}_2\text{O}_3 \text{ (\%)} - 0.547321 \cdot \text{ZrO}_2^* \text{ (\%)} + 0.127883 \cdot \text{Temp. (}^\circ\text{C)} - 0.00105289 \cdot \text{Soaking time (min.)} + 12.2481 \cdot \text{ZnO (\%)} + 2.40802 \cdot \text{CuO (\%)} + 12.0866 \cdot \text{MnO}_2 \text{ (\%)} - 4.14578 \cdot \text{TiO}_2 \text{ (\%)} + 1.25915 \cdot \text{Shrinkage (\%)} + 59.9319 \cdot \rho_a \text{ (g/cm}^3\text{)} - 3.76637 \cdot \rho_r \text{ (\%)} + 1.472 \cdot \text{Abs. (\%)} - 0.198794 \cdot \text{Pd (\%)}$$

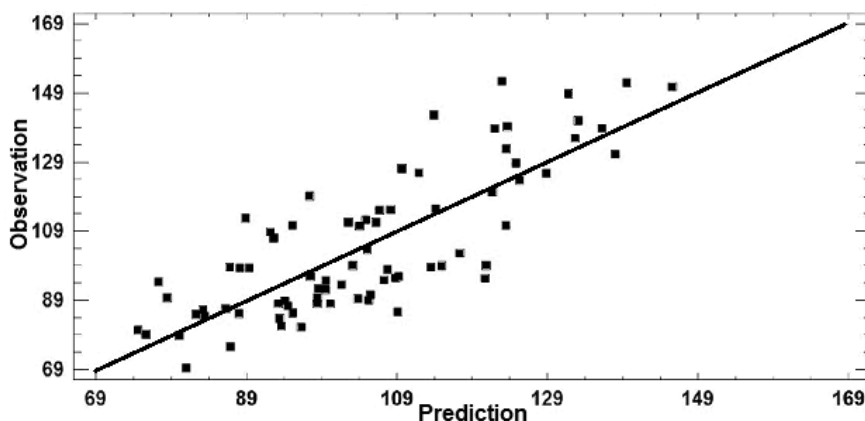


Fig. 7 - The improved compressive strength / Modelul matematic îmbunătățit pentru rezistența mecanică la compresiune.

By introducing the ceramic properties as independent variables will get a higher quality model. It requires a more rigorous control of these properties to improve compressive strength.

**Elasticity model**

Dependent variable: E (GPa)

Independent variables: Al<sub>2</sub>O<sub>3</sub> (%) ZrO<sub>2</sub>\* (%) Temp. (°C), Soaking time (min.) ZnO (%), CuO (%), MnO<sub>2</sub> (%) TiO<sub>2</sub> (%)

**Table 9**

Mathematical factors for elasticity model / Factorii matematice pentru modelul matematic al elasticității

Parameter / Parametru	Estimation / Estimare	Standard Error / Eroarea Standard	P value / Valoarea P
CONSTANT	7.06183	0.922835	0.0000
Al <sub>2</sub> O <sub>3</sub> (%)	-0.00929966	0.00474717	0.0542
ZrO <sub>2</sub> * (%)	-0.00310011	0.00503548	0.5402
Temp. (°C)	-0.00354445	0.000538281	0.0000
Soaking time (min.) <i>Palierul de tratament termic (min.)</i>	-0.000129835	0.000470174	0.7833
ZnO (%)	0.0542653	0.0621537	0.3857
CuO (%)	-0.366047	0.0621537	0.0000
MnO <sub>2</sub> (%)	0.105133	0.0632005	0.1008
TiO <sub>2</sub> (%)	-0.432201	0.0632005	0.0000

R<sup>2</sup> = 75,0661 %

Estimated standard error = 0.325184

Average error modulus = 0.244117

*Elasticity model equation is:*

$$E \text{ (GPa)} = 7.06183 - 0.00929966 \cdot \text{Al}_2\text{O}_3 \text{ (%) } - 0.00310011 \cdot \text{ZrO}_2^* \text{ (%) } - 0.00354445 \cdot \text{Temp. (}^\circ\text{C)} - 0.000129835 \cdot \text{Soaking time (min.)} + 0.0542653 \cdot \text{ZnO (%) } - 0.366047 \cdot \text{CuO (%) } + 0.105133 \cdot \text{MnO}_2 \text{ (%) } - 0.432201 \cdot \text{TiO}_2 \text{ (%)}$$

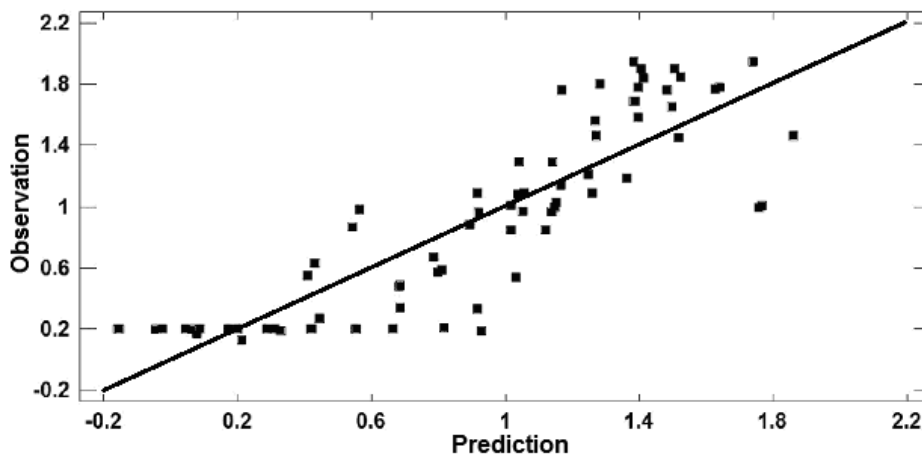


Fig. 8 - Elasticity model / Modelul matematic pentru modulul de elasticitate.

**Groups analysis**

For this analysis it were considered 77 complete data sets, the clustering method was the nearest neighbor and the considered method of calculating the distance was Euclidean quadratic - it calculates the sum of squares of differences of corresponding values.

This procedure created a single group of the 77 data sets. Groups are sets of data with similar characteristics. To form groups, the procedure starts with each set in a separate group, and then it combines the two groups that are closest, then calculating the distance between groups. The closest two groups are then combined into one; this step is repeated until obtaining a single group. The oxide additives variables were removed due to the fact that they were added in very low amounts.



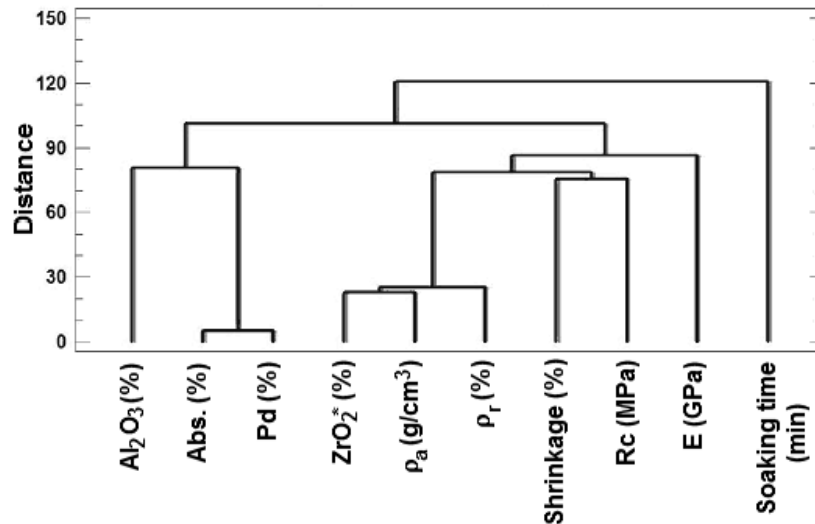


Fig. 9 - Dendrogram / Dendrogramă.

From this groups analysis of analysis, it can be seen that all the variables are grouped according to experimental data [12].

### 3. Conclusions

From done researches we can conclude the following:

- Statistical models designed for ceramic properties are superior to those for mechanical properties.
- The introduction of ceramic properties as independent variables we get a higher quality model for compressive strength.
- Statistical models created are in accordance with existing assertions in this area - are viable, but also offers new insights, are interesting from the point of view of optimizing the compositions as well as the specific properties of ceramics.

### ACKNOWLEDGEMENTS

The experimental results from this study were obtained with the support of Ministry of Labor, Family and Social Protection through the Sectoral Operational Programme Human Resources Development 2007-2013, no. POSDRU/88/1.5/S/60203.

### REFERENCES

1. J. Aitchison, I. R. Dunsmore, Statistical Prediction Analysis, Cambridge University Press, 1980.
2. Ruppert, David, The Elements of Statistical Learning: Data Mining, Inference, and Prediction, Springer, 2009.
3. George W. Snedecor and William G. Cochran, Statistical Methods, 1994.
4. R. Lyman Ott and Micheal T. Longnecker, An Introduction to Statistical Methods and Data Analysis, Duxbury Press, 2000.
5. G. Geoffrey Vining and Scott Kowalski - Statistical Methods for Engineers, Duxbury Press, 2009.
6. R. Lyman Ott, An Introduction to Statistical Methods and Data Analysis: Partial Student Solutions Manual, Duxbury Press, 2009.
7. Barbara Tabachnik, Linda S. Fidell, and Linda Fidell, Using Multivariate Statistics, Allyn & Bacon, 2007.
8. David G. Kleinbaum, Lawrence L. Kupper, Keith and E. Muller, Applied Regression Analysis and Multivariable Methods, Duxbury Press, 2009.
9. Kevin Gurney, An introduction to neural networks, CRC PRESS, 1997.
10. Ron A. Cooper, A. J. Weekes, and Tony J. Weeke, Data, models, and statistical analysis, Rowman & Littlefield Pub. Inc., 1983.
11. Tamraparni Dasu, and Theodore Johnson, Exploratory data mining and data cleaning, Wiley-Interscience, 2003.
12. Ștefania Stoleriu, Alexandru Constantinescu, and Ecaterina Andronescu, Yttria-stabilized ZrO<sub>2</sub>-Al<sub>2</sub>O<sub>3</sub> dense composite ceramics. Obtaining and characterization, Romanian Journal of Materials (2013), **43**(1), 24.
13. Zeno Ghizdăveț, Knowledge discovery in industrial data sets: application. Part I – Analysis, Romanian Journal of Materials (2010), **40**(2), 161.
14. Zeno Ghizdăveț, Knowledge discovery in industrial data sets: application. Part II – Prediction, Romanian Journal of Materials (2011), **41**(1), 73.
15. <http://www.statgraphics.com>, accessed in September 1<sup>st</sup> 2012.

\*\*\*\*\*