STATISTICI UTILIZABILE LA EVALUAREA REZISTENȚEI MECANICE A MATERIALELOR OXIDICE VITROASE: APLICAŢIE PENTRU O STICLĂ DE GEAM

STATISTICAL ASSESSMENTS OF THE MECHANICAL STRENGTH OF VITREOUS OXIDE MATERIALS: APPLICATION FOR A FLAT GLASS

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Both endogenous and exogenous factors influence mechanical strength of oxide glasses. Thus, this property depends on the chemical-structural particularities of the vitreous system, and, also, on the measurement procedures. Due to the influence of random factors, in practice, mechanical strength has distributed values. For its evaluation, different statistical laws can be used.

In the paper, for a flat glass, four types of statistics are tested: the normal law (Gauss-Laplace), the Gamma law, the log-normal law and the Weibull law. Based on the experimental data, a series of statistical indices are calculated, and it is decided which statistical law better model data distribution.

Rezistența mecanică a sticlelor oxidice este determinată atât de factori endogeni, cât și de factori exogeni. Astfel, această proprietate are valori în funcție de particularitățile chimico-structurale ale sistemului vitros, dar și în funcție de condițiile de măsurare. Datorită acțiunii unor factori cu caracter aleatoriu, rezistența mecanică practică prezintă valori distribuite. Pentru evaluarea acesteia, se pot utiliza diferite legi statistice.

În lucrare, pentru o sticlă de geam, sunt testate patru tipuri de statistici: legea normală (Gauss-Laplace), legea Gamma, legea log-normală și legea Weibull. Pe baza datelor experimentale se calculeaza o serie de indicatori statistici și se decide care lege statistică modelează mai bine tipul de repartiție a acestora.

Keywords: *statistical analysis, mechanical strength, flat glass, probability distributions*

1. Introduction

The potential field of use of any material is determined by its main characteristic properties. At the same time, the performance of the material is determined by the value of the leading property (properties). Accurate evaluation of the properties requires *i)* a high precision measurement method and *ii)* adequate statistical processing of experimental data. Among the properties of oxide glasses, mechanical strength has long been studied [1-5]. The attention was motivated by at least two aspects:

- mechanical strength is an important property of vitreous materials, in real life;

- in different structural hypotheses, the theoretical mechanical strength of oxide glass was placed in the range of 7-30GPa; measured values usually ranged within 0.05-0.7GPa. That discrepancy was explained, over time, by the influence of intrinsic and extrinsic factors. The main cause that determines such a drastic decrease in the values of mechanical strength is the presence, postulated by Griffith [1], of some superficial microcracks of nano (sub-nano) metric size. Their appearance is important due to the chemical and dimensional heterogeneity of the component structural entities, at various levels. Therefore, although at the macroscopic level, glass is a homogeneous and isotropic material, at various structural levels the local values of the properties are different. Local tensions can generate superficial microcracks, acting like levers that amplify an applied force. Thus, the catastrophic failure of the glass occurs when applying much smaller external forces than those theoretically expected. There are four groups of factors influencing the practical value of mechanical strength: structural features, genesis route and processing parameters, measurement method and associated influencing factors, statistical method used in processing the experimental data. Image analysis (by Atomic Force Microscopy) of cracks intersecting the free surface of a glass specimen allows to measure stress intensity factors in a quantitative way [6].

In this paper a comparative statistical analysis of the mechanical strength behavior of 50 glass samples was performed; experimental data were processed with various statistical models.

2. Statistical distributions. Statistical indices

The formation and distribution of structural defects in glass, respectively of the microcracks that will generate the rupture, have a probabilistic pattern. For this reason, brittle fracture is a random

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process that generates distributed values for mechanical strength, R, determined on a set of apparently identical samples. This reality requires a statistical evaluation for R.

The statistical variable, X, is associated to the experimental glass mechanical strength, R. The measured values show a certain distribution described by the function $f_X(x,\theta) > 0$, which represents a probability density, defined on a domain $D \in R$. The explicit definition is written as:

$$Prob\{X \in D\} = \int_D f_X(x,\theta)dx = 1 \qquad (2.1)$$

with θ being a parameter that gives specificity to the considered distribution [2].

The probability that the X variable is smaller than a given value x is represented by a function

$$F'_X(x) = f(x,\theta) \qquad (2.2)$$

Using the distribution function, one can calculate the probability that:

$$a \leq X \leq b; Prob\{a \leq X \leq b\} = $$
$$= F_X(b) - F_X(a) \qquad (2.3)$$
$$a \leq X; Prob\{a \leq X\} = F_X(a) =$$
$$= \int_D f_X(x,\theta)dx \qquad (2.4)$$
$$b \geq X; Prob\{b \geq X\} = 1 - F_X(b) \qquad (2.5)$$

where $[a,b] \subset D$.

Essentially, the equations (2.3) - (2.5) give the means to compute the probability that the mechanical resistance R will be placed inside or outside some prescribed value ranges.

Other features of the distributions for continuous variables refer to their degree of dispersion and symmetry. Such indices are represented by moments. Thus, a second order centered theoretical moment, D_2 is defined by the Eq:

$$D_2 = M[X - M(X)]^2 = \int_D x^2 f_X(x)dx -$$
$$- \left(\int_D x f_X(x)dx\right)^2 \qquad (2.6)$$

3. Experimental

3.1. Data processing

The object of the research was a set of 50 samples of indigenous glass. Rectangular shaped samples had their dimensions: length (L) x width (l) x thickness (d) = 0.25 m x 0.038 m x 0.005 m.

The mechanical tensile strength was tested by the 4-point method [1, 5]. The mechanical strength of the samples, R, was calculated with the equation:

$$R = \frac{0.30 \cdot F \cdot a}{l \cdot d^2} \; [MPa] \qquad (3.1)$$

where F denotes failure stress, in N, a – distance between loading points, in m.

The experimental values obtained are presented as an ordered array in Table 1. By processing these data, the statistical values resulted: average = 62.3 MPa (paper [5] give the average of 61.7 MPa), standard deviation = 16.7 MPa, median = 55.6 MPa.

3.2. Calculation of the number of intervals (classes)

3.2.1. Elimination of outliers

In the first stage of evaluating the mechanical resistance, it should be tested that the experimental values do not contain aberrant, erroneous values, called outliers. These outliers occur for various reasons and must be removed in order not to affect the results. Various statistical tests can be applied to eliminate outliers [6-8]; graphical techniques such as, for example, box plots and normal probability plots could, also, help to identify potential outliers. Of these, Grubbs's test is easy to operate with, and it provides numerical results that can be compared with a known threshold (a critical value), thus providing a scientific basis on the decision keep/eliminate the point. The test consists of the following steps:

- ordering experimental data in a limited ascending range of x_{min} up to x_{max};

- statistical indices such as the experimental arithmetic mean, m_{exp}, the experimental standard deviation, σ_{exp}, will be obtained; afterwards, for each value x_i of the array computations will be made according to (3.2):

$$v = \begin{cases} \frac{m_{exp} - x_i}{\sigma_{exp}}, \; if \; x_{min} \; will \; be \; tested \\ \frac{x_i - m_{exp}}{\sigma_{exp}}, if \; x_{max} \; will \; be \; tested \end{cases} \qquad (3.2)$$

- depending on the volume of the database n and on the risk assumed to make an erroneous decision ε, the following comparison shul be made:

$$v = \begin{cases} > \xi_{n;1-\epsilon}, \; the \; value \; should \; be \; removed \\ < \xi_{n;1-\epsilon}, \; the \; value \; should \; be \; preserved \end{cases} \qquad (3.2.1.)$$

Critical values $\xi_{n;1-\epsilon}$ are given in [8].

Table 1

Values of the experimental mechanical strength, R [MPa] / *Valori experimentale ale rezistenței mecanice, R [MPa]*

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Rj [GPa]	39.8	41.0	44.3	45.0	45.0	47.0	47.1	47.4	47.5	48.0	49.4	49.6	49.7	50.1	50.6	50.6	51.1
j	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
Rj [GPa]	52.1	52.3	52.5	53.1	53.6	53.9	54.5	55.6	55.8	57.2	59.4	63.2	63.2	63.5	64.0	64.8	64.8
j	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	-
Rj [GPa]	65.0	67.6	69.0	73.5	75.2	75.3	75.7	77.4	79.3	88.8	88.9	89.0	93.9	95.7	104.5	107.5	-

With the mean value and standard deviation calculated based on the primary data presented in the Table 1, leads to:

$$v(for\ x_{\min}) = \frac{62.3 - 39.8}{167} = 1.35 < 2.97$$

$$v(for\ x_{\max}) = \frac{107.5 - 62.3}{167} = 2.70 < 2.97$$

Note that 2.97 represents the critical value for $\xi_{n;1-\epsilon}$ when n = 50 and ϵ = 0.05. Under these conditions, no value is excluded from the series of experimental values as being an outlier.

3.2.2. Selecting the number of bins (classes)

Various formulas are available for determining the number of classes, *k*; some of them depend on the number of observations, *n*, other being independent on it. For moderate values of *n* (< 50) such as is the given case, Sturges formula is widely used [7,8]:

$$N \cong 1 + 3.33 \cdot lgn = 6.66 \cong 7 \quad (3.3)$$

A simple alternative relationship is written:

$$N = \left[\frac{n}{5}\right] = 10 \quad (3.4)$$

where $[N]$ means the whole part of the number.

The literature also presents other formulas, including ones suitable to larger populations (*n* > 100), having a higher degree of complexity.

As a guide, considering the number of samples used in mechanical strength tests for various types of glass samples, it can be selected *N* = 5÷7 for *n* < 50 and *N* = 6÷10 for 50 < *n* < 100 [9].

Consequently, for a range with equally sized intervals, the size Δx of the class interval is calculated with the simple relation:

$$\Delta x = \frac{x_{max} - x_{min}}{N} \quad (3.5)$$

3.3. Types of statistical distributions

In [7,8] it can be found 40 statistical laws, of which continuous distributions are almost 30. From the most frequently used types of statistical distributions for processing data of materials' strength (including glass) reported in the literature, four such statistical laws (*L*) were chosen for analysis: normal (*N*), log-normal (*Ln*), Weibull (*W*), Gamma (*G*). These are characterized by 1-3 parameters and present a series of connections between them or with other statistical laws. Because in the case of tri-parametric statistics the computation of the values of these parameters cannot be done by analytical means, and the numerical methods are laborious, in the paper the two-parameter forms of the Weibull and Gamma laws will be tested.

4. Testing statistical hypotheses

4.1. The normal law of distribution

Based on the experimental data presented in Table 1, Table 2 is constituted from: class *i* index (range) - col. 1; class range limits for mechanical strength, R, x_{i+1} - x_i, col. 2; the value of the class center, equivalent to the average values of the samples in the interval i, x_{ci}, - col 3; the number of samples in the interval *i*, col 4; frequency in differential form for n_i, $f_i^D = n_i/n = n_i/50$ in column 5; frequency in integral form for n_i, f_i^I, in column 6.

With the data presented in Table 1, the histogram from fig. 1a, respectively the integral distribution of the variable *x* (mechanical strength), fig. 1b, were obtained. The corresponding data are presented in Table 2.

Table 2

Data for constructing the distribution given in Fig. 1 and 2
Datele necesare construirii distribuțiilor din Fig. 1 și 2

i	Δx_i	x_{ci}	n_i	f_i^D	f_i^I
(1)	(2)	(3)	(4)	(5)	(6)
1	30-40	35	1	0.02	0.02
2	40-50	45	12	0.24	0.26
3	50-60	55	15	0.3	0.56
4	60-70	65	9	0.18	0.74
5	70-80	75	6	0.12	0.86
6	80-90	85	3	0.06	0.92
7	90-100	95	2	0.04	0.96
8	100-110	105	2	0.04	1.00

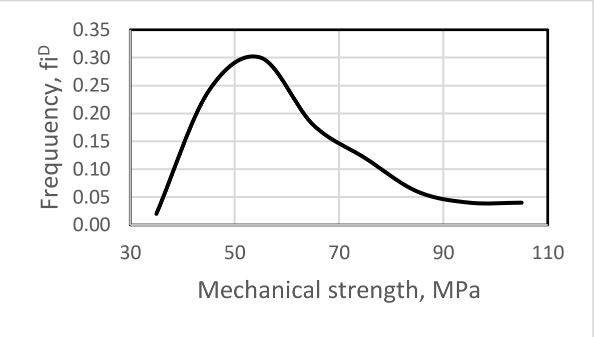

Fig. 1 - *R* differential distribution / *Distribuția diferențială a lui R.*

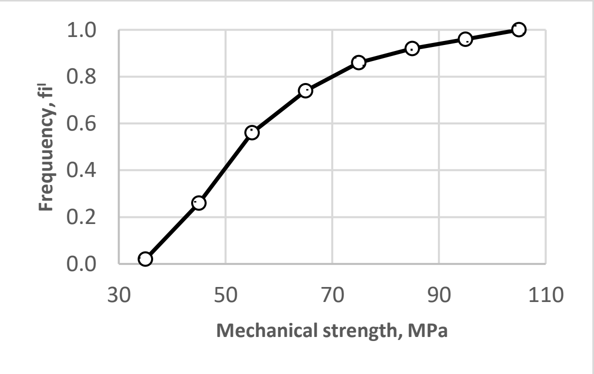

Fig. 2 - Integral distribution of *R* / *Distribuţia integrală a lui R.*

The following statistical indices resulted for this distribution: *average* = 61.8 MPa, σ = 16.42 MPa. It should be noted that, without being identical, the values of the calculated statistical indices are close to those determined using the primary data given in Table 1. Hence the influence of the presentation of primary data on statistical indices.

The histogram of the relative and absolute frequency (cumulated) can predict a certain type of statistical distribution law, but, even in this case, the confirmation of the concordance between the experimental and the theoretical distribution is made by applying different analytical tests, such as the χ^2 test, Kolomogorov test, Massey or Sarkady tests [8]. Because most tests require a relatively large volume of calculations, and some of them have additional restrictions (e.g. the test χ^2 requires that each class will contain at least 5 values, the confirmation of the normality of a distribution can be done using the test of the *r*-order centered moments, D_r, defined by the equation:

$$D_r = \sum_{i=1}^{n} \frac{n_i \cdot \left(x_{c_i} - \sigma_{exp}\right)^r}{n} \quad (4.1)$$

In order to fit a normal distribution, these conditions should be met: $D_3 \approx 0$ and $D_4 \approx 3\sigma^4$. In our case, D_3=4102 ≫ 0 and D_4 = 239032 ≫ 3280, therefore the hypothesis of a normal statistical distribution of the experimental data is rejected.

4.2. Processing experimental data with the Gamma Law

In 1962, E.W. Stacy presented a statistical model called generalized gamma (Γ) [8]. It has a density described by the equation:

$$f(x_i, \theta, k, \nu) = \frac{\nu}{\theta \cdot \Gamma(k)} \cdot \left(\frac{x}{\theta}\right)^{k \cdot \nu - 1} \cdot exp\left[-\left(\frac{x}{\theta}\right)^{\nu}\right] \quad (4.2)$$

where $x \geq 0$, $\theta, k, \nu > 0$, $\Gamma(k) =$
$= \int_0^{\infty} \mathrm{x}^{k-1} \cdot \mathrm{e}^{-x} dx.$

From the tri-parameter Gamma law, a series of other types of distributions of a high generality can be derived. Obtaining the coefficients in Eq. (4.2) requires a laborious numerical calculation; for this reason, the paper discusses the case of a gamma law with two parameters. Considering the relationship (4.2) with ν=1 and 1/θ = a, leads to density:

$$f(x_i, a, k) = \frac{1}{\Gamma(a,k)} \cdot \mathrm{e}^{-ax} \cdot \mathrm{x}^{k-1} \quad (4.3)$$

where $\Gamma(a,k) = \int_0^{\infty} \mathrm{x}^{k-1} \cdot \mathrm{e}^{-ax} dx = \frac{\Gamma(k)}{\mathrm{a}^k}$

In this case, if a random variable *X* follows a Γ distribution, then [7, 8]:

$$M(X) = \frac{k}{a}, B(X) = \frac{k}{a^2}, CV(X) = \frac{1}{\sqrt{k}}, \sqrt{\beta} = = \frac{2}{\sqrt{k}}, \beta_r = \frac{6}{kr} \quad (4.4)$$

Because $CV(X) = \frac{1}{\sqrt{k}} = \frac{\sigma}{\bar{x}}$, σ = 16.4 MPa, it will result *k* = 14.20. Also, *a = k/M(X) =* $k/\bar{x}$ *= 0.23*.

From equation (4.4) it results that $2CV(X) = \sqrt{\beta_1}$; this equality can be used as an empirical criterion to validate Γ. In this case, $\sqrt{\beta_1} = \frac{D_3}{\sigma^3} = \frac{4102}{4411} = 0.93$ and CV = 0.265, so 2CV≠$\sqrt{\beta_1}$, situation in which the hypothesis that the experimental data of mechanical resistance follow a Γ law can be rejected.

4.3. Processing of experimental data with the log-normal law

The log-normal law is an asymmetric distribution, being a function of two parameters: logarithmic arithmetic mean, m_{Ln}, and logarithmic standard deviation, σ_{Ln}. Distribution density $f_{Ln}(x)$ of the *x* variable is given by the equation [7]:

$$f_{Ln}(x) = \frac{1}{x \cdot \sigma_{Ln} \cdot \sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{lnx - m_{Ln}}{\sigma_{Ln}}\right)^2} \quad (5.1)$$

with: $m_{Ln} = \sum_{i=1}^{n} f_i^D \cdot lnx_i$ (5.2)

$$\sigma_{Ln} = \sqrt{\sum_{i=1}^{n} f_i^D \cdot (lnx_i - m_{Ln})^2} \quad (5.3)$$

Setting the parameters for the *Ln* distribution make use of the statistical indices obtained from the database: average m_N, standard deviation, σ_N, and variance V_N. The equations are [7,8]:

$$m_{Ln} = ln\frac{m_N}{\sqrt{1+V_N^2}} \approx lnm_N = 4.13 \quad (5.4)$$

$$\sigma_{Ln} = \sqrt{ln(1+V_N^2)} \approx V_N = 0.27 \quad (5.5)$$

$$Mo_{Ln} = exp(m_{Ln} - \sigma_{Ln}^2) = 57.8 \quad (5.6)$$

$$Me_{Ln} = exp(m_{Ln}) = 63.2 \quad (5.7)$$

Skewness and kurtosis were computed by Eqs. (5.8) and (5.9). Skewness less than 1 indicates that it exists a moderate positive skew (data distribution stretches toward the right tail of the distribution) while kurtosis show a leptokurtic distribution (a positive kurtosis, meaning a higher frequency of data in the central area as compared to the normal distribution).

$$\left(\sqrt{\beta_1}\right)_{Ln} = 3V_N + V_N^3 \cong 0.83 \quad (5.8)$$

$$\left(\beta_2\right)_{Ln} > 3 \quad (5.9)$$

Because the probability distribution of the *Ln* law is asymmetrical, one can conclude that median Me_{Ln} is a better statistical index than the average m_{Ln}.

By making the change of variables $y_i = \frac{lnx_i - m_{Ln}}{\sigma_{Ln}}$, the function *F(x)* and the distribution density $f(x) = \frac{dF(x)}{dx}$, it result the functions corresponding to the normal distribution of the *Y* variable:

$$f_Y(y) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{y^2}{2}} \quad (5.10)$$

$$F_Y(y) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{y} e^{-\frac{y^2}{2}} dy \quad (5.11)$$

The values of these functions are known and given in [8].

N.B. $F_Y(-y) = F_Y(y)$ and $F_X(x_i) \equiv F_Y(y_i)$.

The ordinates of the *Ln* density distribution of the *x* variable, $f_X(x)$, can be obtained by the equation:

$$f_X(x) = \frac{f_Y(y)\cdot\Delta}{x\cdot\sigma_{Ln}} \quad (5.12)$$

where Δ is the class interval ($\Delta = 10$), that should be included to ensure compatibility between f_i^D values from the reference histogram and the computed f_i^D values resulting from the density distribution.

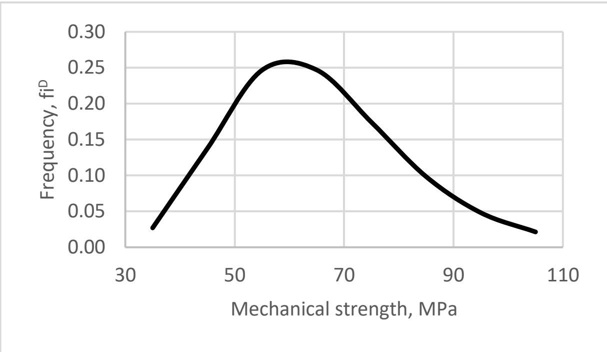

Fig. 3 - Mechanical strength differential distribution according to the log-normal law / *Graficul densității de distribuție a rezistentei mecanice conform legii log-normale*.

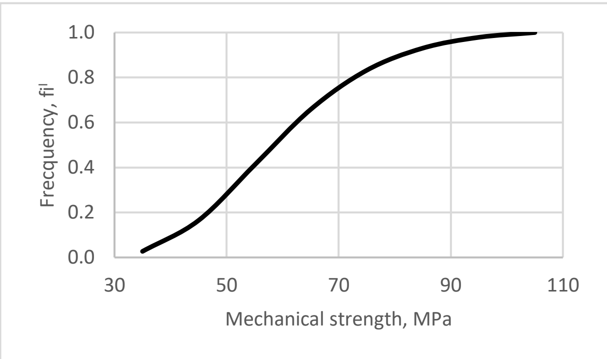

Fig. 4 - Mechanical strength integral distribution according to the log-normal law / *Distribuția integrala a datelor de rezistenta mecanica prelucrate cu Ln law.*

Fig. 3 shows the differential distribution while Fig. 4 gives the integral distribution of mechanical strength data, processed with *Ln* law.

All results show that it exists a moderate departure from the normal distribution; no result, however, supports the hypothesis that the log-normal law fits best the given distribution. Specifically, there are differences between Fig. 1 and Fig. 3 as concerning the frequency amplitude corresponding to various mechanical strengths.

4.4. Processing of experimental data with the Weibull Law

In 1951, W. Weibull proposed a distribution law as a generalization of the exponential law, which applies to the phenomena of fail (destruction) due to complex causes in sustainability tests. After the sixth decade of the last century, Weibull distribution is increasingly used in reliability engineering, but also for evaluating the strength of materials. [8, 9].

Weibull distribution is a statistical law of a wide applicability, characterized by 2 to 5 parameters [8,9]. This gives it a high degree of flexibility. The most used in applications are *W* with 2 or 3 parameters.

W distribution function with 3 parameters, *F(X)*, is:

$$F(x) = 1 - e^{-\left(\frac{x-d}{\theta}\right)^k} \quad (6.1)$$

and the density of the distribution *f(x)* is calculated with the equation:

$$f(x) = \frac{k}{\theta} \cdot \left(\frac{x-d}{\theta}\right)^{k-1} \cdot e^{-\left(\frac{x-d}{\theta}\right)^k} \quad (6.2)$$

k is a shape parameter giving the distribution shape, *θ* – scale parameter, *d* – position parameter, $x \geq d > 0$, $\theta > 0$, $k > 0$.

For $k < 1$ and $d = 0$, *W* distribution is descending; slope is higher when their values are lower. If $k = 1$, *W* law becomes a two-parameter exponential law. For $k > 1$, the distribution curve is bell shaped, being larger the larger *k* is. For $k = 3.25$, *W* law becomes Gauss law; If $k = 2$, *W* law represents Rayleigh distribution; for $d = 0$, *W* law with 3 parameters is transformed into an *W* with 2 parameters.

Average *M(X)* and variance $D^2(X)$ for a *W* law with three parameters can be calculated with the equations:

$$M(X) = d + \theta \cdot \Gamma\left(1 + \frac{1}{k}\right) \quad (6.3)$$

$$D^2(X) = \theta^2 \cdot \left[\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)\right] \quad (6.4)$$

From equations (6.3) and (6.4) the variance coefficient, *V(X)*, cand be obtained. If $d = 0$, the three parameters *W* becomes a two-parameter law. In this case, the mode *Mo* and median *Me* can be calculated with the equations:

$$Mo = \theta \cdot \left(\frac{1}{k}\right)^{\frac{1}{k}} \quad (6.5)$$

$$Me = \theta \cdot (ln2)^{\frac{1}{k}} \quad (6.6)$$

The estimation of the parameters in the case of the *W* distribution can be done graphically (using the W diagram) or by calculation. In the case of the 3-parameter *W* distribution, the analytical calculation is completed with the numerical one [8, 10]. In the case of the distribution with 4 and 5 parameters, the calculation of the parameters becomes particularly tedious. A method of estimating parameters *k* and *θ* of the 2-parameter

W law is based on knowledge of the mean, $\bar{x}$, and of the dispersion, s^2. Thus, the parameter *k* is calculated from the double equation of the coefficient *V(X)* [8]:

$$V(X) = \frac{s}{\bar{x}} = \left[\frac{\Gamma\left(1+\frac{2}{k}\right)}{\Gamma^2\left(1+\frac{1}{k}\right)} - 1\right]^{\frac{1}{2}} \quad (6.7)$$

From Eq. (6.7) the *k* value can be obtained. θ value can be estimated from Eq:

$$\theta = \frac{\bar{x}}{\Gamma\left(1+\frac{1}{k}\right)} \quad (6.8)$$

If the shape parameter, *k*, is known, θ can be calculated by the Eq:

$$\theta = \frac{1}{n}\sum_{i=1}^{n} x_i^k \quad (6.9)$$

A simple and accessible method to obtain the WL parameters is by using the mathematical regression method.

An unimodal strength distribution (6.10):

$$1 - F = e^{-\left(\frac{R}{\theta}\right)^k} \quad (6.11)$$

where $F = \frac{i}{n+1}, i \in [1,8], i \in N, n = 8$

was fitted to a 2-parameter *W* distribution:

$$ln\, ln\frac{1}{1-F} = k \cdot lnR + p \quad (6.12)$$

Note that *R* is the fracture strength, *F* is the cumulative failure probability; the regression constant $p = k \cdot ln\theta$ [3, 8, 10, 11].

Eq. (6.12) can be transformed into a linear one:

$$y = k \cdot x + p \quad (6.13)$$

Regression analysis on mechanical strength R_i, *i* = 1..8, (R_i values are given as x_{ci} in Table 3 for different calculated values, F_i), gives *k* = 2.68, *p* = 11.30; θ = 77.2.

N.B. The result can also be obtained/confirmed graphically using a *W* diagram.

It should be mentioned that the linear dependence given by the relation (6.13), plotted in Fig. 5, has a very high correlation coefficient (R^2 = 0.995). It is thus certified that the bi-parametric *W* law defined by the equation:

$$1 - F = e^{-\left(\frac{R}{77.2}\right)^{2.57}} \quad (6.14)$$

describes very well the distribution of experimental mechanical strength data.

Table 3

Primary data for parameter calculation *k* and θ in Weibull law

Date pentru calculul parametrilor k și θ ai Legii Weibull

i	Δx_i	x_{ci}	F_i	X_i=ln x_{ci}	Y_i=lnln1/(1-F_i)
1	30-40	35	0.11	3.555	-2.15
2	40-50	45	0.22	3.807	-1.39
3	50-60	55	0.33	4.007	-0.92
4	60-70	65	0.44	4.174	-0.55
5	70-80	75	0.55	4.317	-0.23
6	80-90	85	0.67	4.442	0.10
7	90-100	95	0.78	4.554	0.41
8	100-110	105	0.90	4.654	0.83

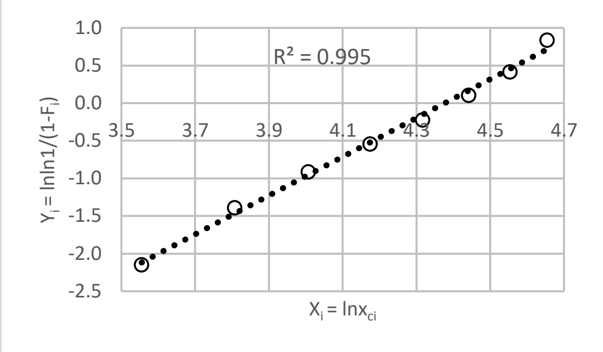

Fig. 5 - Correlation between computed data Y_i with X_i from Table 3 / *Corelarea datelor calculate Y_i cu X_i din Tabelul 3.*

Correspondingly, for the obtained *W*, by using Eq. (6.3) - (6.6), statistical indices were computed (*d* = 0), giving: average $m_w = 77.2 \cdot \Gamma(1.785) = 68.6MP$, standard deviation $s_w = 77.2 \cdot [\Gamma(1.769) - \Gamma^2(1.385)]^{\frac{1}{2}} = 28.9MPa,$ mode $Mo_w = 77.2 \cdot 0.5^{0.39} = 63.4MPa,$ median $Me_w = 77.2 \cdot 0.693^{0.39} = 66.9MPa.$

5.Conclusions

Griffith microcracks size distribution at glass surface is stochastic in nature; therefore, the mechanical strength of flat glass should obey probabilistic laws.

Probability distributions are statistical functions that give the expected outcomes of possible values for given data. In this case, probability distributions give the opportunity to predict the likelihood of failure at a given applied stress. Four of the most used statistical functions for modeling the experimental results were evaluated: normal law, gamma, log-normal and, respectively, Weibull with 2 parameters.

The object of this evaluation was the flat glass mechanical tensile strength, that was tested by the 4-point method. Mechanical strength determinations were made on 50 samples of glass plates, and the experimental average and standard deviation are in a good agreement with other scientific results [5].

Data processing by statistical tests showed that the hypotheses of the normal and bi-parametric gamma distribution are rejected. Also, the computed log-normal distribution fail to comply with the frequency distribution of the experimental data.

Based on the regression analysis (R^2 = 0.995) when processing data in Section 4.4, the most conclusive result was obtained with the Weibull law. This result confirms other worldwide published findings, stating that Weibull law fits best mechanical strength data distribution for a series of materials, including glass [5, 11, 12]. The reason for this – somehow expected – outcome is the known versatility of the Weibull law.

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