ANALIZA COMPARATIVĂ A CONSTANTELOR ELASTICE INGINEREȘTI ALE STRATIFICATELOR COMPOZITE

COMPARATIVE ANALYSIS OF THE ENGINEERING CONSTANTS OF COMPOSITE LAMINATES

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Composite structures subjected to complex states of stresses require tailored strength and stiffness characteristics in certain directions, while the properties of the unidirectional lamina are disproportioned and unequal in different directions. Therefore, a multi-layered composite is needed, with various stacking sequences of elementary laminas and different fibre orientations. An accurate prediction of the elastic engineering constants is essential, in order to evaluate the stiffness properties of composite laminates.

The paper presents a comparative analysis of the in-plane elastic engineering constants of continuous fibre reinforced laminated composites, pointing out the influencing parameters of the stiffness properties of multi-layered composites. Three types of laminates, made of composite laminas reinforced with S glass fibres or E glass fibres, embedded in an epoxy resin, with different geometric arrangements of laminas are analysed: an asymmetric general laminate, an anti-symmetric angle-ply and a symmetric angle-ply laminate. The obtained results, illustrated by the graphical distributions of the engineering constants, reveal superior stiffness properties in case of particular laminates.

Structurile compozite solicitate la stări complexe de tensiuni necesită caracteristici de rezistență și rigiditate dirijate după anumite direcții, în timp ce proprietățile unei lamele armate unidirecțional sunt disproporționate și inegale pe direcții diferite. Din acest motiv, este necesară alcătuirea unor structuri compozite stratificate cu anumite succesiuni ale straturilor elementare și diferite orientări ale fibrelor. O cunoaștere precisă a constantelor elastice inginerești este esențială, pentru o evaluare a caracteristicilor de rigiditate ale stratificatelor compozite.

Lucrarea prezintă o analiză comparativă a constantelor elastice inginerești în planul unor stratificate compozite armate cu fibre continue, evidențiind parametrii ce influențează caracteristicile de rigiditate ale acestora. Sunt analizate trei tipuri de stratificate, realizate din lamele compozite armate cu fibre de sticlă S sau fibre de sticlă E, înglobate în rășină epoxidică, cu diferite configurații geometrice: un stratificat general asimetric, un stratificat unghiular antisimetric și un stratificat unghiular simetric. Rezultatele obținute, reprezentate prin distribuția grafică a constantelor inginerești, evidențiază caracteristici de rigiditate ridicate în cazul stratificatelor particulare.

Keywords: *in-plane engineering constants, composite laminates, fibre orientation, glass fibres*

1. Introduction

Fibre reinforced polymeric (FRP) composites are widely used structural materials, having strong directional properties, mainly in case of continuous fibres. Due to their remarkable properties, they are utilised in various applications, such as construction, automotive, marine, electrical, aerospace, oil and gas industries [1, 2].

The plurality of fibrous composites advantages, such as the outstanding mechanical properties, high strength related to the lightweight, tailoring of performance, flexibility in design, corrosion resistance, the improved fatigue strength, have led to an increasing utilisation in load-bearing structures [1].

A composite lamina is the simplest element of a composite material, an elementary layer or a building block for multi-layered or composite laminates. It consists of a thin ply of composite material, generally realized of parallel fibres embedded in a matrix [3]. The stiffness of unidirectional lamina in transverse direction is very poor compared to the longitudinal one, because the matrix properties dominate [4]. Therefore, a laminate is formed by stacking two or more laminas with different fibre orientation angles, in the direction of the lamina thickness [5].

The glass fibres, which are the most frequently utilised reinforcing fibres due to their relative low cost, attain high strength as a result of the low number and size of defects on the fibre surface. For structural FRP composites use, the most commonly types of glass fibres are S-glass and E-glass, in case of elements with load bearing structural, strengthening and electrical purposes [6-9]. Although glass fibres develop high tensile strength, chemical resistance and good impact

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strength, they have low fatigue resistance, poor adhesion with polymers and relatively low modulus of elasticity [10-13].

Epoxy resins are thermosets with excellent processability and good electrical insulating properties, widely used in structural adhesives, surface coatings, but also as a matrix for high-strength engineering composites and structural laminates [14].

An accurate prediction of the in-plane elastic engineering constants is needed, in order to characterize the stiffness behaviour of laminated composites. The elastic properties of multi-layered composites are influenced by the constituent material characteristics, orientation and location of the individual unidirectional laminas [15, 16].

2. Stiffness properties of laminated structures made of FRP composites

In order to apply the lamination theory predictions for the macroscopic evaluation of the stiffness properties of multi-layered composites, several aspects regarding micromechanics of orthotropic lamina are needed. Therefore, the specially orthotropic lamina, illustrated in Figure 1, is defined as an elementary thin layer of composite material, with the principal material axes (1, 2) identical with the general system of axes (x, y).

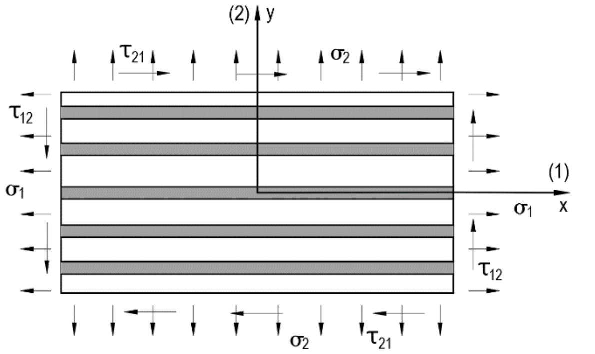


Fig. 1 - The specially orthotropic lamina / *Lamela ortotropă specială.*

The analysis of composite laminates is realized in a similar manner as in strength of materials, by establishing a constitutive relationship between stress and strain, according to Hooke's law [17]. The constitutive equations which describe the linear-elastic response of the specially orthotropic lamina is given in Eq. (1a) [18]:

$$\{\sigma\} = [Q]\{\varepsilon\}, \tag{1a}$$

or in the expanded form in Eq. (1b):

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}, \tag{1b}$$

where:

$[Q]$ is the reduced stiffness matrix;

σ_1, σ_2, τ_{12} are the in-plane stress components with respect to the principal material directions;

ε_1, ε_2, γ_{12} are the in-plane strain components with respect to the principal material directions.

The elements Q_{ij} of the stiffness matrix are expressed in terms of the engineering constants of the lamina, with respect to principal material directions, as follows [19]:

$$Q_{11} = \frac{E_1}{1-\nu_{12}\cdot\nu_{21}}$$
$$Q_{12} = \frac{\nu_{21}\cdot E_1}{1-\nu_{12}\cdot\nu_{21}} = \frac{\nu_{12}\cdot E_2}{1-\nu_{12}\cdot\nu_{21}} \tag{2}$$
$$Q_{22} = \frac{E_2}{1-\nu_{12}\cdot\nu_{21}} \qquad Q_{66} = G_{12},$$

where:

E_1, E_2, G_{12} are the axial and shear elastic moduli along the principal material axes;

ν_{12}, ν_{21} are the Poisson's ratios referring to the axes 1 and 2.

The generally orthotropic lamina, shown in Figure 2, also called the off-axis unidirectional lamina, makes a fibre orientation angle θ with respect to the arbitrary system of axes (x, y).

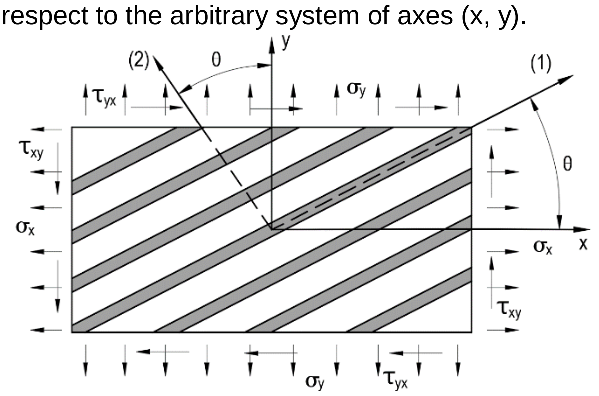


Fig. 2 - The generally orthotropic lamina / *Lamela ortotropă generală.*

The constitutive equation which describes the relationship between stresses and strains for a generally orthotropic lamina, is given by the condensed form in Eq. (3a) [18]:

$$\{\sigma\} = [\overline{Q}]\{\varepsilon\}, \tag{3a}$$

or in the expanded form in Eq. (3b):

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}, \tag{3b}$$

where:

$[\overline{Q}]$ is the transformed reduced stiffness matrix;

σ_x, σ_y, τ_{xy} are the in-plane stress components along the global reference axes;

ε_x, ε_y, γ_{xy} are the in-plane strain components along the global reference axes.

The terms $\overline{Q}_{ij}$ of the transformed reduced stiffness matrix are expressed in Eq. (4), as follows [20]

$$\begin{aligned}
\overline{Q}_{11} &= Q_{11}c^4 + 2(Q_{12}+2Q_{66})c^2s^2 + Q_{22}s^4\\
\overline{Q}_{12} &= (Q_{11}+Q_{22}-4Q_{66})c^2s^2 + Q_{12}(c^4+s^4)\\
\overline{Q}_{22} &= Q_{11}s^4 + 2(Q_{12}+2Q_{66})c^2s^2 + Q_{22}c^4\\
\overline{Q}_{16} &= (Q_{11}-Q_{12}-2Q_{66})c^3s +\\
&+ (Q_{12}-Q_{22}+2Q_{66})s^3c\\
\overline{Q}_{26} &= (Q_{11}-Q_{12}-2Q_{66})cs^3 + (Q_{12}-Q_{22}+2Q_{66})sc^3\\
\overline{Q}_{66} &= (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})c^2s^2 + Q_{66}(c^4+s^4),
\end{aligned} \tag{4}$$

where: $c = \cos\theta$, $s = \sin\theta$.

Because the fibre contribution to the transverse stiffness of the unidirectional lamina is less compared to that in the longitudinal direction, it is necessary to add laminas with various orientations in the thickness direction, in order to satisfy the design requirements [21]. Therefore, laminates are conceived and designed from unidirectional composite laminas as to improve the directional properties and satisfy the design needs of specific composite structures [1, 22].

The general laminates consist of an arbitrary number of layers stacked and bonded together, oriented in the desired direction [21, 23]. The angle-ply laminate is a particular case of the specially orthotropic laminate, obtained by alternating equal number of layers with +θ and –θ fibre orientation angles, with same thickness and material properties [17, 24].

Symmetric laminates have identical layers in thickness, fibre orientation and material, symmetrically disposed with respect to the middle plane [24]. Reversely, the anti-symmetric laminates are identified by having plies arranged in an anti-symmetrical way about the mid-surface [17].

The geometrical characteristics for a general laminated composite having *n* layers and *2H* thickness are illustrated in Figure 3:

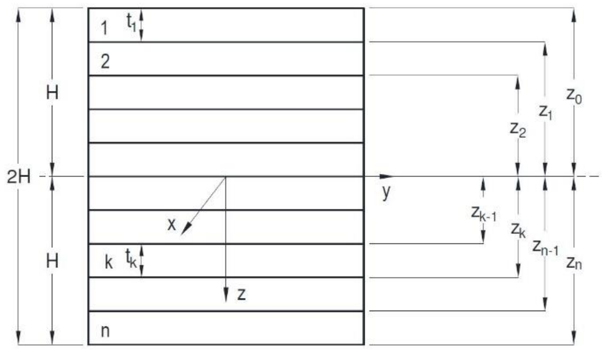


Fig. 3 - Geometrical characteristics of a n-layered laminate
Caracteristici geometrice ale unui stratificat cu n straturi.

The constitutive equation which describes the relationship between stresses and strains in a multi-layered laminate is written for an elementary layer *k* of a generally orthotropic lamina, given by the condensed form in Eq. (5a) [24]:

$$\{\sigma\}_k = \left[\overline{Q}\right]_k \{\varepsilon\}_k , \tag{5a}$$

or in the expanded form in Eq. (5b):

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}_k = \begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}\end{bmatrix}_k \begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}_k , \tag{5b}$$

The resultant in-plane forces (N_x, N_y, N_{xy}) and moments (M_x, M_y, M_{xy}) acting on the middle plane of the laminated composite is shown in Figure 4.

The force-deformation and moment-deformation relations of multi-layered laminates are described by Eq. (6a) and (6b):

$$\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\end{Bmatrix} = \begin{bmatrix}A_{11} & A_{12} & A_{16}\\ A_{12} & A_{22} & A_{26}\\ A_{16} & A_{26} & A_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{Bmatrix}k_x\\ k_y\\ k_{xy}\end{Bmatrix}, \tag{6a}$$

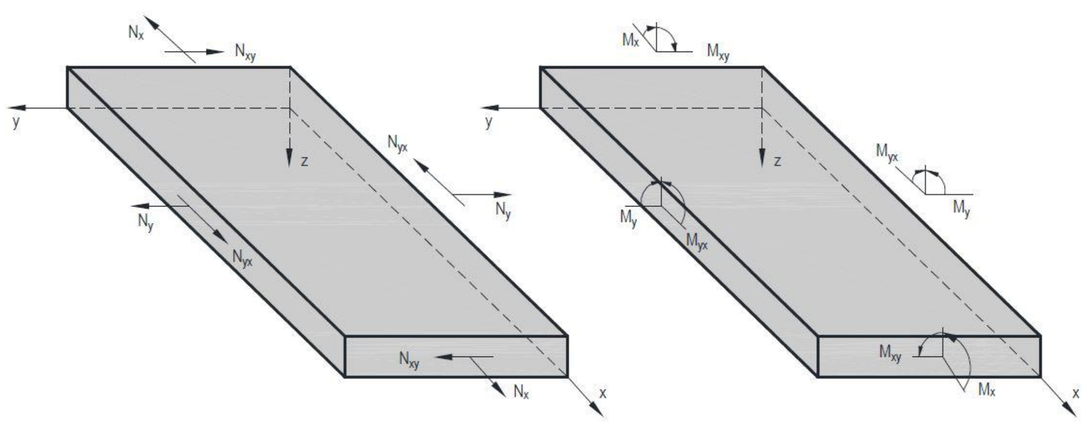


Fig. 4 - In-plane forces and moments per unit width / *Forțe și momente pe unitatea de lățime.*

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix}, \tag{6b}$$

where:

($\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0$) represents the laminate mid-plane strains;

(k_x, k_y, k_{xy}) represents the laminate curvatures;

The contribution of the geometric configuration of the fibres orientation and the stacking sequence on the laminate elastic behaviour is expressed through the matrix terms [A], [B], [D], involved in the force-deformation and moment-deformation relations and discussed below [15, 25].

The extensional stiffness matrix or *the in-plane stiffness matrix [A]* relates the resultant in-plane forces to the in-plane strains and it can be evaluated with Eq. (7a) or (7b), as follows [10, 26]:

$$A_{ij} = \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \cdot \left(z_k - z_{k-1}\right), \tag{7a}$$

$$[A] = \left[\sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \int_{z_{k-1}}^{z_k} dz \right] = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \tag{7b}$$

where:

z_k, z_{k-1} are the coordinates to the bottom and to the top of the *k* layer.

The bending-stretching coupling matrix [B] importance is referred to the coupling effect between forces and moments to the middle plane strains and middle plane curvatures. Matrix [B] is determined according to Eq. (8a) or (8b) [5, 24]:

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \cdot \left(z_k^2 - z_{k-1}^2\right), \tag{8a}$$

$$[B] = \left[\sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \int_{z_{k-1}}^{z_k} z\,dz \right] = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \tag{8b}$$

The bending stiffness matrix [D] relates the resultant bending moments to the plate curvatures [5, 24] and it is expressed as follows:

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \cdot \left(z_k^3 - z_{k-1}^3\right), \tag{9a}$$

$$[D] = \left[\sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \int_{z_{k-1}}^{z_k} z^2 dz \right] = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \tag{9b}$$

The various coupling effects in laminated composites can be minimized or eliminated through suitable choices of the laminas stacking sequences. The mid-plane symmetry arrangements eliminate the bending-stretching coupling [15]. In case of angle-ply laminate, the D_{16} and D_{26} terms can be diminished by increasing the number of unidirectional laminas.

The in-plane engineering constants of laminated composites are determined for the given conditions in Eq. (10), such that the non-zero stresses corresponds to the elastic constants in the needed directions.

$$\begin{aligned} E_x &= \frac{\sigma_x}{\varepsilon_x^0}, \text{ when } \sigma_x \neq 0,\ \sigma_y = 0, \tau_{xy} = 0, \\ E_y &= \frac{\sigma_y}{\varepsilon_y^0}, \text{ when } \sigma_y \neq 0,\ \sigma_x = 0, \tau_{xy} = 0, \\ G_{xy} &= \frac{\tau_{xy}}{\gamma_{xy}^0}, \text{ when } \tau_{xy} \neq 0,\ \sigma_x = 0, \sigma_y = 0, \\ \nu_{xy} &= -\frac{\varepsilon_y^0}{\varepsilon_x^0}, \text{ when } \sigma_x \neq 0,\ \sigma_y = 0, \tau_{xy} = 0, \\ \nu_{yx} &= -\frac{\varepsilon_x^0}{\varepsilon_y^0}, \text{ when } \sigma_y \neq 0,\ \sigma_x = 0, \tau_{xy} = 0. \end{aligned} \tag{10}$$

In case of the in-plane Poisson's ratio, ν_{xy}, both longitudinal strain ε_x^0 and transverse strain ε_y^0 must be written in terms of an applied stress in the x-direction, while in case of the in-plane Poisson's ratio, ν_{yx}, both longitudinal strain ε_y^0 and the transverse strain ε_x^0 will be evaluated in terms of an applied stress in the y-direction.

The development of the explicit relations, which describe the in-plane elastic characteristics of multi-layered composites, is related to [A], [B], [D] matrices, taking account of stress-strain relationships [27].

$$E_x = \frac{\begin{vmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{vmatrix}}{\begin{vmatrix} A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{vmatrix}} \cdot \frac{1}{2H} \tag{11a}$$

$$E_y = \frac{\begin{vmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{vmatrix}}{\begin{vmatrix} A_{11} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{16} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{66} & D_{16} & D_{26} & D_{66} \end{vmatrix}} \cdot \frac{1}{2H} \tag{11b}$$

$$G_{xy} = \frac{\begin{vmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{vmatrix}}{\begin{vmatrix} A_{11} & A_{12} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & B_{12} & B_{22} & B_{26} \\ B_{11} & B_{12} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & D_{16} & D_{26} & D_{66} \end{vmatrix}} \cdot \frac{1}{2H} \tag{11c}$$

$$\nu_{xy} = \frac{\begin{vmatrix} A_{12} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{66} & D_{16} & D_{26} & D_{66} \end{vmatrix}}{\begin{vmatrix} A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{vmatrix}} \tag{11d}$$

$$\nu_{yx} = \frac{\begin{vmatrix} A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{vmatrix}}{\begin{vmatrix} A_{11} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{16} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{66} & D_{16} & D_{26} & D_{66} \end{vmatrix}} \tag{11e}$$

where: 2H represents the total thickness of the laminate.

3. Case studies

For an accurate prediction of the elastic engineering constants for multi-layered composites, a comparative analysis is performed according to the classical lamination theory. The study is carried out for three different geometric arrangements of laminas, with 4 layers and the same ply thickness, t=0.4 mm, resulting in the following configurations: a general laminate $\left[\pm\theta/(\theta)_2\right]$, an anti-symmetric angle-ply laminate $\left[(\pm\theta)_2\right]$ and a symmetric angle-ply laminate $\left[\pm\theta\right]_s$. The analysis is materialized through the variations of the in-plane elastic engineering constants with respect to fiber orientation angles, for three selected fibre volume fractions. The composite material properties are given in Table 1 [19], the laminates being made alternatively of S glass or E glass fibres and epoxy resin.

The implication of the [A], [B], [D] matrices on the elastic behaviour of laminates, influenced by the orientation of fibres and stacking sequences, can be observed in Table 2, for both types of glass fibres.

The differences between the analysed laminates can be firstly observed from the corresponding [A], [B], [D] matrices, involved subsequently in the engineering constants graphics. It can be noticed that for the general laminate, $\left[\pm\theta/(\theta)_2\right]$, the in-plane extensional and the shear coupling occurs, because of the nonzero A_{16} and A_{26} terms. As expected, both $\left[(\pm\theta)_2\right]$ and $\left[\pm\theta\right]_s$ angle-ply configurations are specially orthotropic laminates, proven by $A_{16} = A_{26} = 0$. Moreover, the requirement that $D_{16} = D_{26} = 0$ has no need to be fulfilled in case of specially orthotropic multi-layered composites, since they are distinguished by the in-plane response and not by the bending response. However, the D_{16} and

Table 1

Mechanical characteristics of constituent materials / *Caracteristicile mecanice ale materialelor constituente*

Constituents of the composite laminas *Materialele constituente ale lamelei compozite*	Longitudinal Young's modulus *Modulul de elasticitate longitudinal* [GPa]	Poisson's ratio *Coeficientul lui Poisson*
S glass fibres / *fibre de sticlă S*	85.5	0.22
E glass fibres / *fibre de sticlă E*	72.4	0.22
epoxy resin matrix / *matrice din rășină epoxidică*	4.1	0.4

Table 2

[A], [B], [D] stiffness matrices for θ= 45º and V_f = 70% / *Matricile de rigiditate [A], [B], [D] pentru θ= 45º și V_f = 70%*

S glass fibres and epoxy resin / *fibre de sticlă S și matrice epoxidică*			
	[A]	[B]	[D]
$[\pm\theta/(\theta)_2]$	$\begin{bmatrix} 29.54 & 19.25 & 7.43 \\ 19.25 & 29.54 & 7.43 \\ 7.43 & 7.43 & 20.30 \end{bmatrix}$ [GPa·mm]	$\begin{bmatrix} 0 & 0 & 1.12 \\ 0 & 0 & 1.12 \\ 1.12 & 1.12 & 0 \end{bmatrix}$ [GPa·mm^2]	$\begin{bmatrix} 3.55 & 2.31 & 1.56 \\ 2.31 & 3.55 & 1.56 \\ 1.56 & 1.56 & 2.44 \end{bmatrix}$ [GPa·mm^3]
$[(\pm\theta)_2]$	$\begin{bmatrix} 29.54 & 19.25 & 0 \\ 19.25 & 29.54 & 0 \\ 0 & 0 & 20.30 \end{bmatrix}$ [GPa·mm]	$\begin{bmatrix} 0 & 0 & -2.23 \\ 0 & 0 & -2.23 \\ -2.23 & -2.23 & 0 \end{bmatrix}$ [GPa·mm^2]	$\begin{bmatrix} 3.55 & 2.31 & 0 \\ 2.31 & 3.55 & 0 \\ 0 & 0 & 2.44 \end{bmatrix}$ [GPa·mm^3]
$[\pm\theta]_S$	$\begin{bmatrix} 29.54 & 19.25 & 0 \\ 19.25 & 29.54 & 0 \\ 0 & 0 & 20.30 \end{bmatrix}$ [GPa·mm]	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ [GPa·mm^2]	$\begin{bmatrix} 3.55 & 2.31 & 1.34 \\ 2.31 & 3.55 & 1.34 \\ 1.34 & 1.34 & 2.44 \end{bmatrix}$ [GPa·mm^3]
E glass fibres and epoxy resin / *fibre de sticlă E și matrice epoxidică*			
	[A]	[B]	[D]
$[\pm\theta/(\theta)_2]$	$\begin{bmatrix} 26.61 & 16.53 & 6.08 \\ 16.53 & 26.61 & 6.08 \\ 6.08 & 6.08 & 17.51 \end{bmatrix}$ [GPa·mm]	$\begin{bmatrix} 0 & 0 & 0.91 \\ 0 & 0 & 0.91 \\ 0.91 & 0.91 & 0 \end{bmatrix}$ [GPa·mm^2]	$\begin{bmatrix} 3.19 & 1.98 & 1.28 \\ 1.98 & 3.19 & 1.28 \\ 1.28 & 1.28 & 2.10 \end{bmatrix}$ [GPa·mm^3]
$[(\pm\theta)_2]$	$\begin{bmatrix} 26.61 & 16.53 & 0 \\ 16.53 & 26.61 & 0 \\ 0 & 0 & 17.51 \end{bmatrix}$ [GPa·mm]	$\begin{bmatrix} 0 & 0 & -1.83 \\ 0 & 0 & -1.83 \\ -1.83 & -1.83 & 0 \end{bmatrix}$ [GPa·mm^2]	$\begin{bmatrix} 3.19 & 1.98 & 0 \\ 1.98 & 3.19 & 0 \\ 0 & 0 & 2.10 \end{bmatrix}$ [GPa·mm^3]
$[\pm\theta]_S$	$\begin{bmatrix} 26.61 & 16.53 & 0 \\ 16.53 & 26.61 & 0 \\ 0 & 0 & 17.51 \end{bmatrix}$ [GPa·mm]	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ [GPa·mm^2]	$\begin{bmatrix} 3.19 & 1.98 & 1.10 \\ 1.98 & 3.19 & 1.10 \\ 1.10 & 1.10 & 2.10 \end{bmatrix}$ [GPa·mm^3]

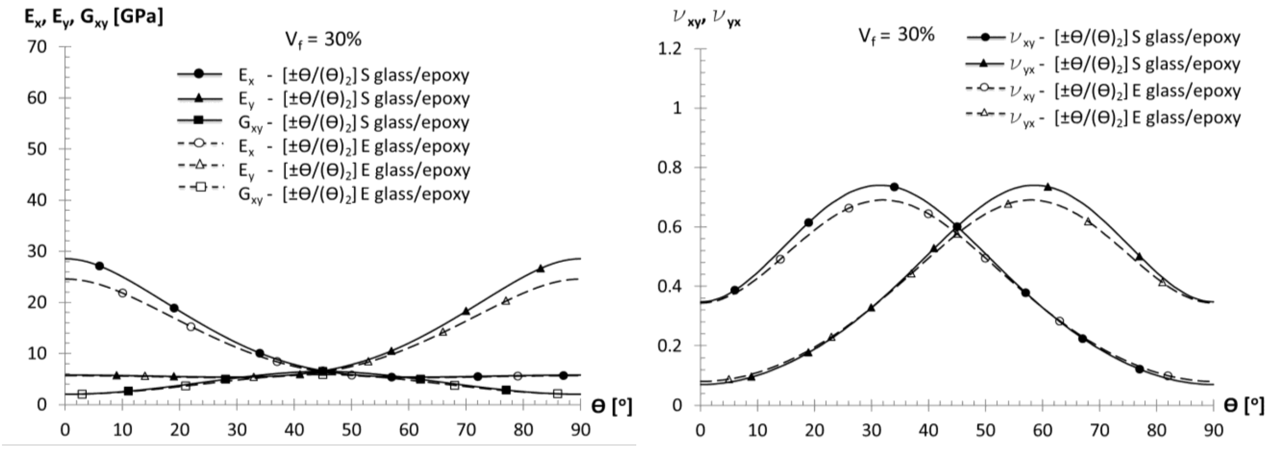
D_{26} zero terms are noticed for the anti-symmetric angle-ply case, while for the symmetric angle-ply laminate, the bending - torsion coupling occurs. Because of the stacking sequence symmetry in case of $[\pm\theta]_S$ angle-ply laminate, the [B] matrix is zero, meaning that there is no coupling between in-plane extensional and bending response.

Figures 5-7 illustrate the graphical representation of the in-plane engineering constants variation for the studied configurations of laminates, selected for three relevant fibre volume fractions (V_f = 30%; 50%; 70%), both for S glass fibres and epoxy resin, and also for E glass fibres and epoxy resin composites.

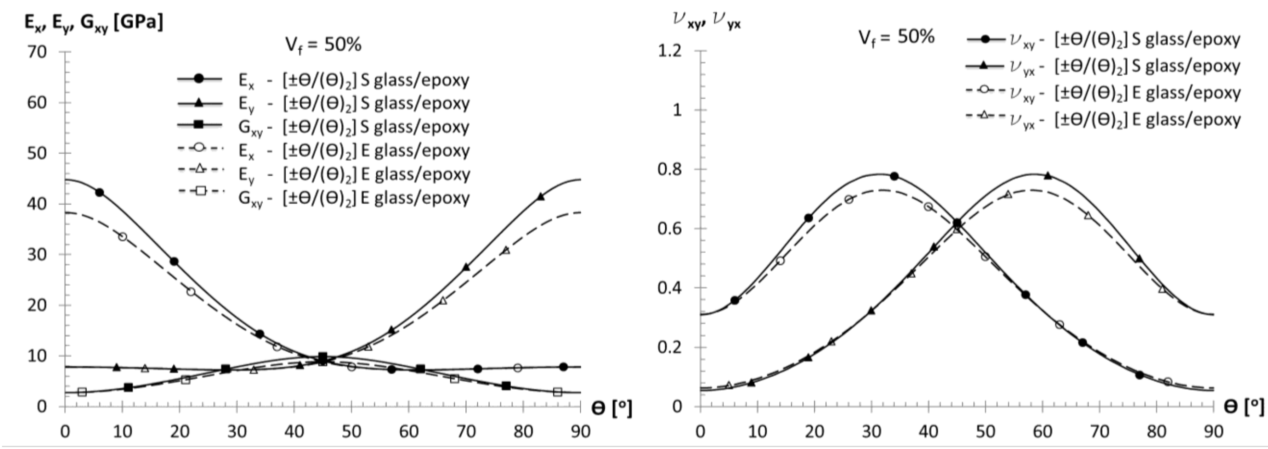
As it can be seen from the graphics, for certain intervals of fibre orientation angles, the engineering constants distributions show increased elastic properties for the multi-layered composites made of S glass fibres compared with E glass fibres laminates.

The contribution of fibres in achieving higher stiffness is demonstrated by the longitudinal and transverse moduli variations. The in-plane elastic behaviour of laminates is balanced at a fibre orientation angle of 45º, where both axial and shear modulus show approximately the same values. Moreover, at same fibre orientation, the laminated composites achieve high shear stiffness. The maximum values of the Poisson's ratios suggest that the multi-layered composites enable additionally elastic responses, compared with the traditional material.

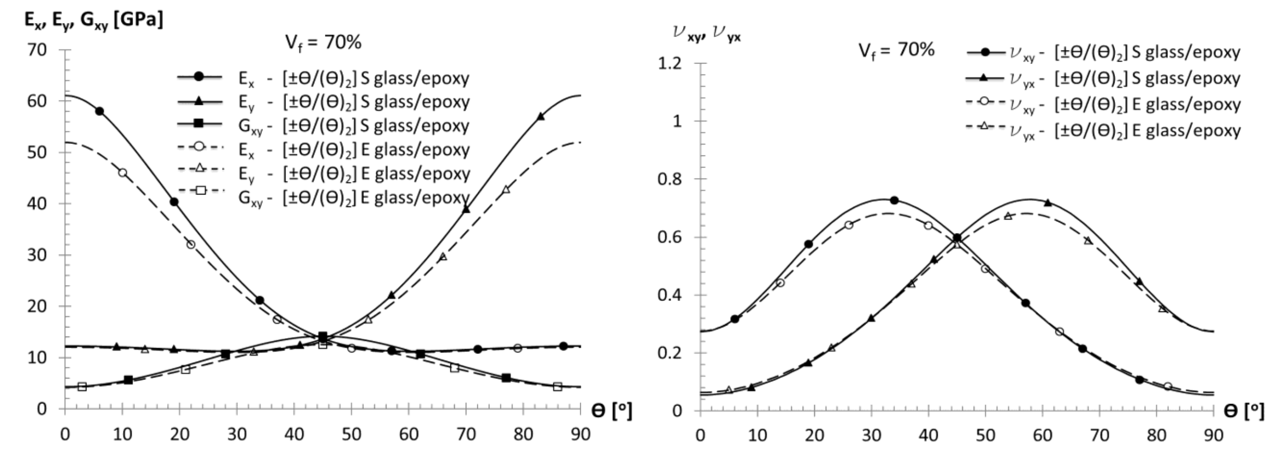
Since for fibre orientations of 0º and 90º, the in-plane engineering constants of the analysed multi-layered composites are the same, the differences between the laminates are noticeable



a). V_f = 30%

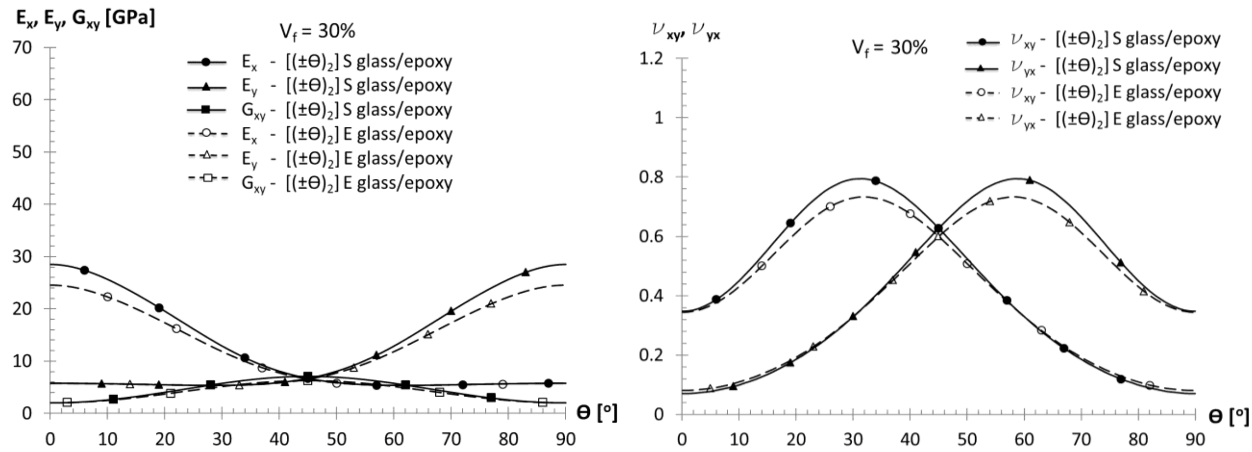


b). V_f = 50%

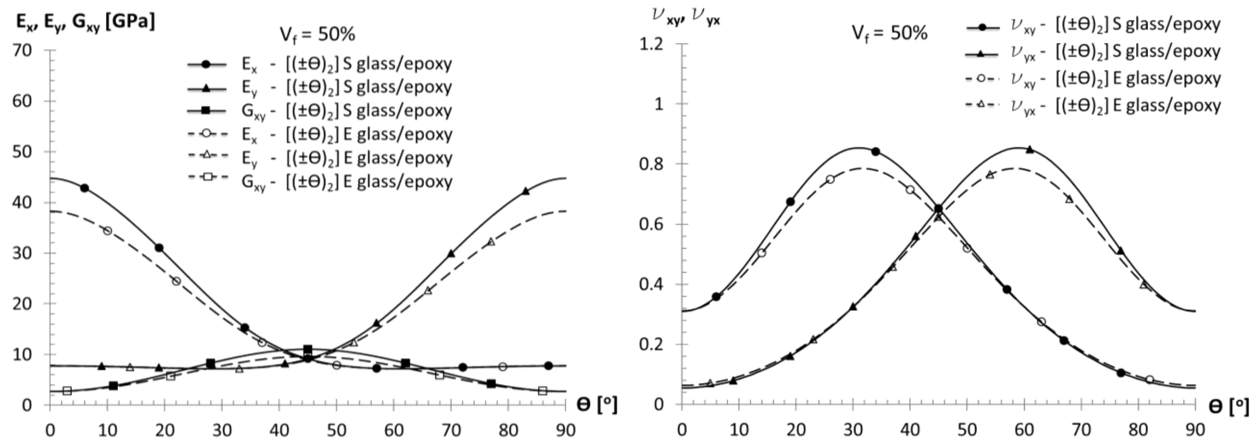


c). V_f = 70%

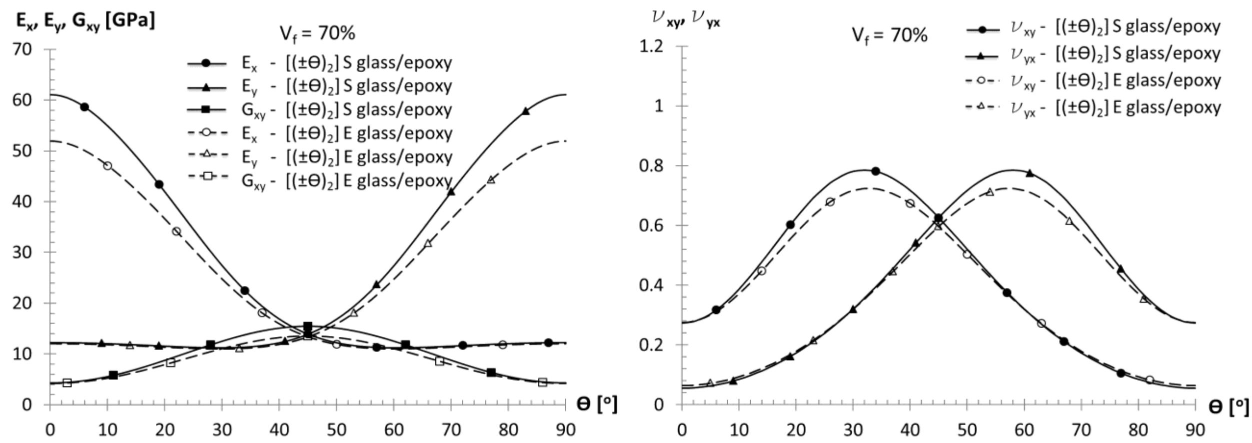
Fig. 5 - The in-plane elastic engineering constants for general laminates / *Constante elastice inginerești în planul stratificatelor generale*



a). $V_f = 30\%$

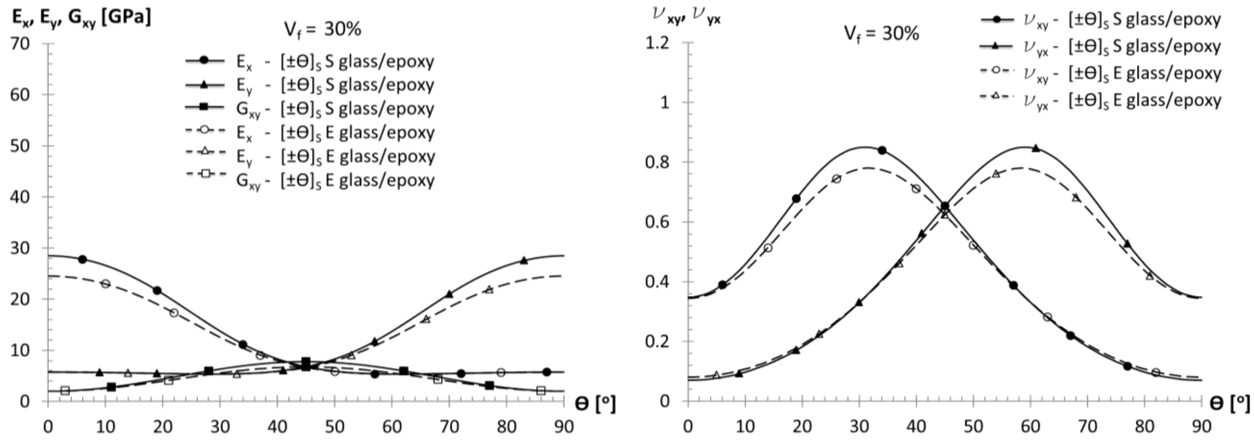


b). $V_f = 50\%$

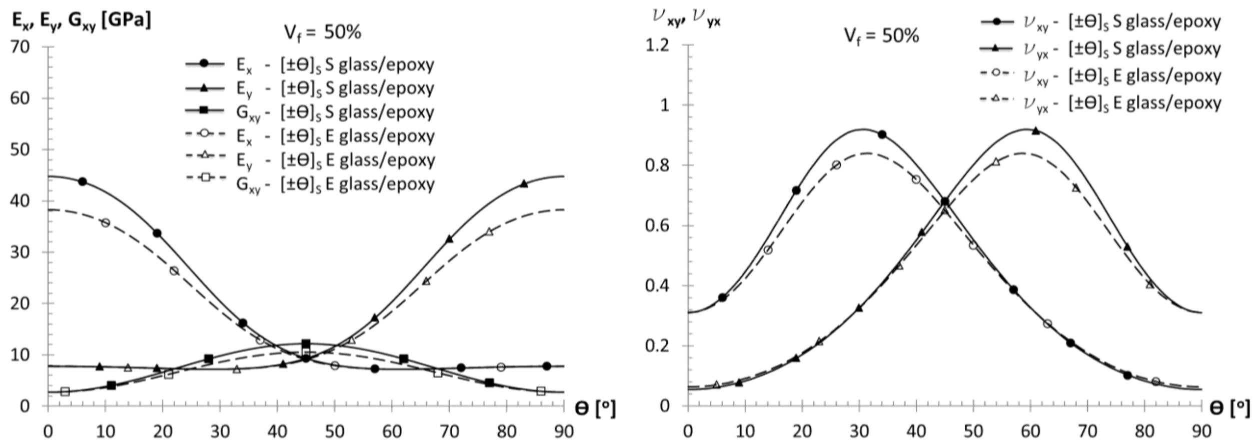


c). $V_f = 70\%$

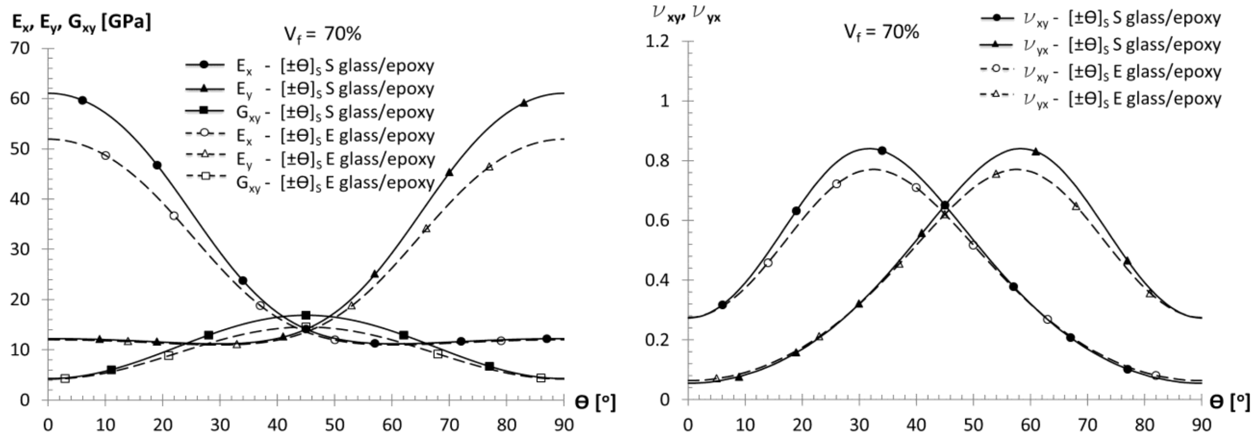
Fig. 6 - The in-plane elastic engineering constants for anti-symmetric angle-ply laminates */ Constante elastice inginerești în planul stratificatelor unghiulare antisimetrice.*



a). $V_f = 30\%$



b). $V_f = 50\%$



c). $V_f = 70\%$

Fig. 7 - The in-plane elastic engineering constants for symmetric angle-ply laminates / *Constante elastice inginerești în planul stratificatelor unghiulare simetrice .*

for certain intervals of fibre orientation angles. In case of the longitudinal modulus E_x, angle-ply laminates are stiffer than the general configuration laminate, for fibre orientation angles up to 5º to 40º, while for transverse modulus E_y, for $\theta = 50 - 85^o$. Significant differences occur in case of the shear modulus G_{xy}, for fibre orientations of 10º to 80º. For the Poisson's ratio ν_{xy}, the stiffening effect is visible for 15-50º fibre orientations interval and, as expected, in case of ν_{yx}, for fibre orientation angles of 40º to 75º.

As shown in Figures 5-7, the stiffness behaviour of laminated composites is improved with increasing fibre volume fraction, but also through suitable choices of the stacking sequences. Therefore, the elastic performance of particular laminates compared with general laminates is demonstrated for the specified intervals of fibre orientations.

The symmetric laminates show superior elastic characteristics than the anti-symmetric or asymmetric multi-layered structures.

Therefore, the tailoring benefit of composite laminates to achieve the required stiffness properties in the desired direction, can be obtained by selecting the right influencing parameters, such as constituents of the composite, fibre volume fractions, fibre orientations, number of plies and stacking sequences.

4. Conclusions

A comparative analysis of the elastic engineering constants of three types of laminates has been performed. The influence of the elementary layers' parameters on the elastic properties of composite laminates is demonstrated by graphical distributions of the in-plane engineering constants. The importance of using the appropriate fiber orientation angle and stacking sequence or the adequate composite material is crucial when it is necessary to meet the stiffness requirements for the design of a composite structure.

The variation curves illustrate improved elastic behaviour of particular laminates compared with general laminates, introduced by the stacking sequence symmetry. Interchanging this important parameter when designing multi-layered composites, will affect its response to bending or may influence the coupling effects. Symmetric laminates are therefore preferred for most practical situations.

In terms of material aspects, the laminated composites realized of S glass fibres reach the maximum values of the elastic engineering constants, for certain fibre orientations.

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